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ANNUAL REPORT
ON

STUDIES OF PATTERN-PERCEIVING AND LEARNING NETWORKS

Issued Under

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INFORMATION SYSTEMS BRANCH
MATHEMATICAL SCIENCES DIVISION
OFFICE OF NAVAL RESEARCH

28 February 1961

By
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COMMITTEE ON MATHEMATICAL BIOLOGY
THE UNIVERSITY OF CHICAGO
CHICAGO, ILLINOIS

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1. Introduction

This report consists of a summary of progress achieved under the present contract, a list of the publications containing a full statement of these results, and a collection of extracts from published reports, progress reports, and class notes used in teaching. These notes present additional topics and preliminary results not discussed in publications, together with notes on various mechanisms which appear to be important ingredients of perception.

Given a roomful of perfectly simulated neurones, retinas, modifiable synapses, and the like, one would not know when to say they were perceiving without knowing something about how perception takes place, and research on perceptual machines has therefore utilized models containing various ingredients believed to be present in natural perceptual organization. While there is no guarantee that artificial perception must proceed just like natural perception, examples of the most successful artificial endeavors tend to support this contention, at least at the level of general principles. However, we still have a lot more to learn about integrative principles in perception.

In any successful endeavor we must at some stage develop a rigorous treatment establishing the possibilities and limitations of particular modes of synthesis or modification of logical functions or network response characteristics. These problems are being studied widely. But in natural systems and in the artificial systems that work the best we find new configurations, goals, and operators being generated by the systems themselves. The difficulties and novelties are generally in the organization and integration of these emergent structures. Unless we first understand the principles of organization, we cannot tell just what relation something like a variable threshold or a particular computer routine, for example, might bear to perception. Or perhaps an entirely different aspect of system functioning is the one most relevant to perception.

Workers in artificial perception have already contributed to our understanding of the ways in which the component process fit together. The present notes contain some additional psychological information which has not yet been fully utilized by these workers. This information seems very suggestive of practical applications, but it is not available in a form which has proved readily accessible to those whose interests lie along mathematical lines. It therefore seems that one of the most valuable functions of the present report might be to summarize some of this information for the use of those persons to whom this progress report is distributed. This information serves also as a statement of problems to which research under the present contract is addressed, and is thus a prerequisite for understanding the purpose of the previously published results and the aims of studies now underway.

2. Summary of Published Research Covered by this Report and Work in Progress.

Perceptual devices which have been extensively studied function through the processing of sensory inputs, as in networks analogous to those in the visual pathway, or through the construction of a network which produces an automatically regulated function of input values, or which carries out certain measurements upon inputs and makes decisions based on them. The present report discusses problems of the integrated functioning of such devices, and presents behavioral material which shows some of the things a machine would have to do in order for us to say it was perceiving.

The difficulty in studying the problems emphasized in this report is that one must know the right questions to ask, but this requires having some idea of the aspects of the biological or mechanical system which are relevant to perception, and one cannot know these without knowing what questions to ask. Therefore it seemed worthwhile to study model systems satisfying the conditions (1) that their structure could be completely known, and (2) that they could be naturally described in the same terms that are used to speak of characteristically perceptual processes. Within these systems one might learn how to ask fruitful questions and at the same time learn which further properties of truly perceiving systems are missing in the model system.

In the example of such a model system which has been investigated during the period covered by this report, information is regarded as being represented in the form of vectors (such as modes of oscillation of a complicated system, or columns in a matrix representing the structure of a network), which may be resolved into components in various coordinate systems. These systems represent various points of view from which the information may be regarded, and some of the information in each system may be elicited by a probabilistic mechanism in a way which mimics the mathematics of the process of observation in quantum mechanics although the model has nothing to do with quantum physics. After presenting one conceivable instance for the realization of such a scheme, I shall mention some of the parallels with psychological problems.

This instance concerns networks proposed by Marcus Goodall, now at MIT, for the purpose of carrying out transformations which are of value in the statistical analysis of data. One such network is designed to transform a set of correlated inputs into a set of independent outputs, or what is equivalent, to express a set of input functions as a set of linear combinations of orthonormal output functions. This can be done by a feedback network in which each negative feedback gradually becomes proportional to the correlation between the activity of the output which is its source and the input which is its destination. This is to say that the feedback transfer functions become the Fourier coefficients in the expansion of the inputs in terms of an orthonormal set of outputs. The possible orthogonal outputs corresponding to a given input differ by unitary transformations and are determined by the inputs and by the initial state of the net. If we modify this situation by forcing the outputs to assume the values of some set of orthogonal functions, then the feedback transfer matrix will converge to the expansion coefficients of the inputs in terms of the given orthogonal set. Then we may express vectors in terms of orthonormal coordinate systems which have been partly determined by the initial state of the observing system -- in a very crude sense by the observer's "point of view." Such a network would be a special case of the hypothetical neural network discussed by Milner in his paper on cell assemblies in the brain. A related network performs a transformation of the inputs to the principal axis system of their covariance matrix. This transformation is known to be useful in reducing the amount of data which must be considered in a complicated situation, and may sometimes be used in communications systems to reduce the number of channels required to carry a given number of messages, for this procedure may be used to find a few linear combinations of inputs which carry the bulk of the input information. Thus the particular coordinate system in which the covariance matrix is diagonal assumes special significance; and in the model being investigated under the present contract, it was assumed for purposes of investigation that the coordinate systems representing various points of view might be those in which certain operators became diagonal. How may the model be said to resemble psychological processes?

One important requirement for brain function is that each element of information should contain partial representations of many other elements and schemata for their interconnection. It is well known that in the formalism adopted for the present model, each frame of reference can contain complex exponential factors which make no difference in that frame, but which encode information about potentialities in other frames of reference. Certain perturbation properties have analogues in Gestalt constancies, while structural properties of composite systems bear some resemblance to Gestalt properties of composite stimuli and their parts. Purely as a formal device -- just to see what it looked like -- it was supposed that the quantum-like projections onto subspaces with probabilities proportional to the squared magnitudes of expansion coefficients was accomplished by means of a process described by Wiener, in which the vectors are averaged with samples of shot noise and the results used in a fashion which happens to make the probabilities come out right. It turned out unexpectedly that elements in this part of the model bore analogies to some fundamental processes of instinctive behavior (in which actions are elicited by the combination of specific stimuli and a certain level of drive excitation, and in which superpositions of response modes may occur in a way analogous to the model) and processes of learning theory (in which the expansion coefficients were like habit strengths and the noise level like drive level; or an entirely different way in which the model could mimic statistical learning theory of Bush, Estes, etc.). Finally, the model bore some resemblances to stages in the development and clarification of ideas as they come into awareness.

Present efforts are devoted to investigating particular aspects of these and related processes of perception which show promise of application to artificial perceptual systems. Plans for the immediate future call for the application of these ideas as a tool for studying some of the problems outlined in the remainder of this report, such as the integration of schemata of action, and the construction and representation of perceptual geometry on the basis of various types of sensation. Some of these plans may be summarized as follows:

People studying artificial perception often try to make a list of features of behavior and then try to think of a physiological structure that can be described in the same way. This is not necessarily the best way to proceed, but it is a reasonable thing to try. The usual list reads: various stimuli and responses can become connected. Therefore people look for neural elements which can become anatomically or functionally connected to other elements. Then they simulate these elements electronically. The present studies are based upon a longer list of about fifteen features of behavior, which includes the usual list and fourteen other features consistent with the usual list. If the longer list were shown to a group of electrical engineers who were not told that it had anything to do with psychology, they would probably almost all say that it was a list of the ways in which network outputs can be analyzed into combinations of natural modes. The contents of this list and the ways in which the responses occur will be explained elsewhere. It turns out that they can have a number of properties which are particularly biological. Moreover, if the information occurs in the form of these responses, then information in one part of the brain could be transmitted to another part solely by transmitting individual frequencies, and nothing more complicated, and this is just what nerve pathways are good at transmitting. It would also follow without further mechanisms that an incoming signal could excite a response consisting of a complicated pattern of activity in many elements, and at the same time not excite a different pattern involving exactly the same elements.

In addition to various other biological features which will be described elsewhere, there are some potentially far reaching implications of an assumption that the sensory inputs are allowed to excite natural modes of linear networks (dendrites??) which then excite all-or-none firing which transmits information to new networks. An interesting book by Jean Nicod, Foundations of Geometry and Induction, tells what kind of geometry of the physical world could be discovered by organisms with limited senses but perfect logical facilities. For instance, as one would expect, an organism limited to an external sense who can tell whether any two sensed particulars are the same or different, and can remember the order in which stimuli occur as he moves about, can determine the connectivity of the space in which he is located. More surprisingly, an organism with only a kinaesthetic sense (in which two movements are entirely alike or else are regarded as entirely different) plus one external reference point (like a buzzer at one point of space), can on this basis alone (this summary has omitted no detail!) define straight lines, parallel translations, congruence of two motions, and thus all of Euclidean geometry. Finally, an organism with a generalized type of vision which apprehends points in a three-dimensional sensitive visual volume, can build Euclidean spaces of places, of objects, and of views (in which the points are various equivalence classes). In the space of views an individual point is itself a space having an ascertainable geometry. But if the vision is achieved by projecting points upon a two-dimensional surface as we do, the space of objects disappears, while the objects become embedded within a space of views in a certain sense not possible for three-dimensional vision. In certain subtle ways the two-dimensional vision turns out to be better than the three-dimensional vision.

Now Nicod's methods are not restricted to organisms moving around in three-dimensional space, which is what he considered. They apply just as well to points moving around in function space, or points in a phase space, the dimensions of which correspond to all the different muscles and nerves involved in an action. Nicod's "three-dimensional" and "two-dimensional" cases apply to any number of dimensions in which you either apprehend position or else position of the projection on the unit sphere, respectively. In particular, if inputs are expanded in terms of natural modes, transformations in the external world become mapped into isomorphic transformations of the coefficients of the modes. The spatial transformations of inputs which have to be learned may be learned from crude expansions into just a few modes, so that the vector space can have very low dimension. These ideas are all related to the question of how a spatial order comes to permeate all sorts of activity of the infant. They may help explain how geometrical relations in the external world can be represented in terms of muscles and movements. These ideas are all relevant to Piaget's problems, outlined in previous reports, and they were inspired primarily by observations of ethologists concerning the organization of behavior.

Even if no animal uses these methods, there may still be two useful technological applications:

(1) A pattern recognizer, instead of measuring various attributes or establishing connections between "neurones," might work by letting the pattern excite modes of oscillation -- such as modes in a wave guide or acoustic modes in a tube, or the natural oscillations of a network. These would respond, for example, with some of the same swirling or slashing modes as did Piaget's children in trying to draw squares. The strengths of the various modes could be measured by sending in the inputs at constantly rising frequency starting

at zero. As each cutoff frequency or resonant frequency is reached the corresponding mode is transmitted down the tube, and the increments of power are measured. If these numbers (frequencies and powers) were then fed into some analyzing device (logical network, Perceptron, etc.), the whole combination might be more like what happens in the brain than is anything now in existence. A particular version of this model can provide a realization of the vector space model of perception which has been studied.

(2) A machine for performing skilled actions, instead of being based upon the steady-state response of some set of levers or tubes, with various oscillations and instabilities as things to get rid of, might instead exploit the oscillations themselves as the vehicle for the action. This would be clearer if there were space in the present proposal to explain how the ideas of the normal modes came from studies of instinctive behavior, but that explanation will be presented elsewhere. (The basic idea is that they correspond to the inherent action patterns out of which an animal synthesizes its responses to stimuli. They can be excited by random transients and run their course even in the absence of the external stimuli for which they are designed. The natural modes under investigation behave similarly, in about fourteen specific ways. Such things could not help occurring in animals with networks, and must have let to overt behavior. Evolution either suppressed this feature or exploited it. Since it sounds like the way animals behave, the latter might be suspected.)

In the case of a simple linear network described by an equation $By = x$, where B is a matrix of differential operators, x is a forcing vector, and y is a response vector, this means building responses out of solutions to the homogeneous equation $By = 0$. These can be put together in ways which correspond to the fourteen points inspired by ethology. To consider a simple example, let us contrast two ways one might build a simplified model horse (ignoring for present purposes whatever way a real horse is actually constructed). (a) For each gait one builds a network that makes four (or more) motors operate in prescribed phase relations. For one gait one might let leg A be connected to leg B, but inhibit leg C, while C is connected via a certain number of delay elements (like synaptic delays) to D, and so on. Then one inserts switches to select which network is to operate. (b) One writes down the phase relations of the legs in the various gaits (for the moment each leg does simple harmonic motion, but one can add more dimensions to get more realistic motions). One then writes down a set of numbers expressing the average frequencies (i.e., the speeds at which the horse goes) corresponding to the gaits -- say, 1, 2, 3, 4. Then one simply has to find a matrix B (s), where s is complex frequency, whose rows are orthogonal to the first vector of phases when $s = j$ ($= \sqrt{-1}$), to the second vector when $s = 2j$, to the third when $s = 3j$, and to the fourth when $s = 4j$. Then one must build a network with this matrix. One then has a single system -- not four systems connected with a selector switch -- having no parts which may be interpreted as inhibitory connections, delay units, etc., which when it goes slowly walks, when it goes faster trots, when still faster gallops, etc.

Much of the proposed research will be devoted to clarifying problems related to the above ideas and trying to determine whether they would really work in a practical way.

Publications During the Period Covered by This Report

1. "Networks for Pattern Perception," in Proceedings of the National Electronics Conference, Vol. 15 (Chicago, 1959), pp. 357-369.

2. "A Suggested Model for Information Representation in a Computer That Perceives, Learns, and Reasons," in Proceedings of the Western Joint Computer Conference (San Francisco, 1960), pp. 151-164.

3. "Networks Which Realize a Model for Information Representation," in Symposium on the Principles of Self-Organization (University of Illinois, June, 1960), edited by H. von Foerster and G. Zopf. (In press)

4. "Some Biological Ideas," Bionics Symposium, Dayton, September, 1960. WADD Technical Report 60-600, pp. 495-497.

Other Relevant Publications

1. In Search of the Fundamental Units of Perception: An Outline, to appear as Air Force Office of Scientific Research Technical Report, AFOSR TN 60-622. (Revision of unpublished outline of lectures given in a psychology seminar at Northwestern University, 1959.)

2. "An Approach to Computers That Perceive, Learn, and Reason," in Proceedings of the Western Joint Computer Conference (San Francisco, 1959), pp. 181-186. (ASTIA No. AD - 213 855.)

3. Some Biological Ideas

(Remarks from the USAF Bionics Symposium, Dayton, Ohio, Sept., 1960)

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After listening to the interesting papers which have been presented during the last few days, I was struck by some ideas which I should like to sketch in part.

The subtitle of this Bionics Symposium is "Living Prototypes -- The Key to New Technology." We have had sessions entitled "The Life Sciences in Bionics," "Analysis of Biological Principles," and "Mechanical Realization of the Higher Functions of Living Systems." But I believe that some of the most characteristically biological features have not been mentioned.

Although I concede that we do not yet know enough to recreate these features or even to describe them in adequate detail, I feel that unless we view this lack with regret and keep these features in view as part of our goal, we shall be much the poorer.

If the Russians knew how to build truly biological machines, we would spare no pains to learn their secrets. We would gladly master their foreign language and interpret their repellant phraseology in our terms so that we might take advantage of their knowledge. But I believe that engineers are not taking these pains in regard to much of the knowledge gained by modern psychology. I'd like to list in barest outline a few of the biological features which differ from present machine design. Since these exist at a prelogical stage of thought development, for which we do not have an adequately descriptive language, the full force of these points can be felt only by reference to numerous biological examples.

1. Present designs for perceptual machines subject inputs to a number of transformations in order to obtain a useful result. Real biological systems largely fabricate their experience out of their own constructions. The elemental behavior scheme seems to be largely one in which stimuli act by switching the organism from one of its existing modes of behavior to another. These modes are inherent or derived through processes that obviously use processed inputs like the machines, but the direction of emphasis is from inside to outside in biology, outside to inside in most machine design.

2. In present design one program or function may be added onto another, or may be included in it, while in typical biological patterns of thought and action one schema may thoroughly permeate another in a way which may be grasped through numerous examples, but which we do not yet know how to characterize precisely. The difference feels, purely metaphorically, like the difference between the direct sum of two matrices and their Kronecker product; or one might feel that one structure is somehow built out of ingredients bearing the mark of another, in the way perhaps that a quotient group tells how one group is built up out of another. When the infant sucks the nipple, his hands may open and close in the same rhythm as his lips; and guided by a host of similar occurrences we may say that the schema of grasping and taking in is extended to movements of the hand, and comes ultimately to permeate more and more movements. This is one reason why living organisms, in contrast to machines, can perform the same purposive and skilled action with any available set of effectors.

3. In present design of learning devices a typical elemental mechanism is the association of elements of information regarded as given. In biological thought and action we typically find in addition that objects and processes become defined and articulated by being incorporated into ever-enlarging schemata in the way hinted at above.

4. Present machine design may lead to a single device that transforms all patterns of a large class to size- or orientation-invariant form. Biological systems may in addition develop such transformations in a piecemeal way. For instance, a child learns some of his spatial abilities by looking at and grasping various objects here, then there, then another place. This is much less efficient for any particular ability than the former approach (which is also common in living organisms), but it can lead to highly integrated total patterns of behavior as the grasping schemata gradually permeate increasingly varied and extensive schemata.

5. In present design the receptor system is generally separate from the effector system. In living organisms it is a corollary of the discussion of the piecemeal elaboration of perceptual schemata through the piece by piece integration of schemata of action, that a machine that really can be said to perceive will not just perceive, but will have potentialities for performing skilled actions, at least in rudimentary form.

Now what techniques might be used to implement these principles? Are they just as elusive and ill-defined? Without trying to be at all precise or complete, we believe that in the living organism they involve such things as some sort of activating impulses, scanning, servomechanical processes which tend to conserve useful or interesting states, and hierarchical organizations of all the above principles and structures. These are all conceptually familiar to the engineer. In other words, if we can clarify the meaning of principles of behavioral organization such as the five which I have mentioned, we have tools now to implement them.

If we think only in terms of the customary trends of emphasis, we shall continue to deal brilliantly with the mechanical manipulations of symbols for our ideas, but the mechanical synthesis of the thought itself will seem supernatural. I believe that appreciation of some of the above-mentioned biological patterns of emphasis makes this synthesis seem remarkable, but not supernatural. Although some of these points may seem like describing the same things in two different ways, it can be shown that the two points of view actually can lead to different research objectives.

To employ this biological pattern of emphasis would be merely silly in the design of machines with typical present-day goals; and to abandon these goals and the much more efficient machine approach to them would be unthinkable. To aim at duplicating living organisms would be quixotic. But somewhere in between I believe there is a level of great flexibility and versatility of perception and skilled action which engineers are going to achieve through the use of these biological methods. They are altogether compatible with all present techniques, and in fact would depend upon them. Their rudiments may be clearly seen in a number of programs which are in working order at present. I hope that more people of varied experience will become enthusiastic about keeping these goals in mind as they go about doing the kinds of things reported at this symposium.

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[1]A few of the relevant references may be found in the bibliographies of my papers in the Proceedings of the 1959 Western Joint Computer Conference and the Univ. of Ill., Symposium on the Principles of Self-Organization (1960), (in press), and some discussion in the Proc., 1960, W.J.C.C.

4. From <u>5th Quarterly Progress Report</u>, Office of Naval Research Information Systems Branch Contract No. Nonr 2121(17) NR 049-148 Nov. 30, 1960

(The following is an expansion of the five points made at the Bionics Symposium, September, 1960.)

Here is a hint of the way in which the general ideas I expressed at the Bionics Symposium could be related to perceptual systems. I'll start by listing a handful of ideas from Piaget which I feel must be basic ingredients in the desired processes. These exemplify the five points: (1) Not just useful transformations of inputs, but fabrication of perception out of its own constructions, which are then combined and modified by learning; (2) Permeation of one schema of action by another, so the same skilled action can be performed by any available set of effectors and the same meaningful patterns perceived in diverse circumstances; (3,4) emphasis on the incorporation of more and more objects and processes into schemata rather than just association of preformed entities: (5) perception involves action -- trying things out on its own.

(Piaget's ideas in this section come mainly from <u>The Origins of Intelligence in Children</u>. The other two books on this stage (out of his two dozen or so) are <u>The Construction of Reality in the Child</u> and <u>Play, Dreams and Imitation in Childhood</u>.)

In Piaget's investigations of how the child comes to perceive spatially and to recognize and manipulate objects an extensive study was made of how these notions of space and objects develop from sensori-motor phenomena. I'm not trying to put down here the details and explanations of how the child proceeds, but only trying to indicate the sorts of processes emphasized. He starts with reflexes, or in otherwords, with preformed automatisms. The emphasis is on the need for repetition and practice to achieve normal function. Activity is augmented by its own use, and the reflex tends to "incorporate into itself" every object capable of exciting it. That is, these objects become incorporated into integrated schemata of action -- it's not just patchwork. Two directions of emphasis are this "assimilation" of new objects to the schemata, and a converse process of "accommodation" of the schemata to the environmental conditions. This "generalizing assimilation" leads to complex schemata -- the child sucks his thumb, then can carry things to his mouth, then forms a kind of "mouth space" which becomes interconnected with other "momentary spaces" induced by particular actions. This generalization -- fitting all sorts of things into the same schema -- isn't to be thought of as confusion or mere lack of differentiation, an evil to be improved upon. Rather, these acts are the stuff upon which other action schemata are built. (It takes lots of examples to give content to these statements.) Stimulation of a schema leads the child to repeat its stimulation by all sorts of things. This "reproductive assimilation" leads to the "stabilization" of what has been incorporated into the schema. Once a schema is stabilized, the child uses it in the recognition of objects through the excitation of the schema ("recognitory assimilation"). The schema is by then in "equilibrium" so recognitory assimilation doesn't lead to more reproductive assimilation and further change.

Generalizing assimilation, according to Piaget, occurs in the early stages not so much by applying previously learned patterns in a systematic way to new stimuli, but rather, by repetition in which the activity is reproduced in its original form, and during the course of which novelties are encountered by chance.

For instance, the child may grasp a piece of his blanket during the course of opening and closing his fist, and if this occurs often enough, he acts as if he reached for the blanket in order to grasp. Gradually, grasping for the sake of grasping becomes converted into grasping objects. This is another instance of how behavior develops from inside to outside--the putting to good use of a pre-adapted action in the child's repertoire.

At first there are separate spaces of grasping, sucking, etc. When an object passes from hand to mouth, it ceases to exist as a grasping object, and the grasping space disappears. But at the same time a sucking object is momentarily created, and a sucking space. Thus a space and an object exist only as long as a reflex is active.

Sometimes the evocation of a schema for one kind of perception is considered by Piaget to be incompatible with the evocation of a different sort of perception. As an example of such incompatible perceptual operations, Piaget (in The Child's Conception of Space) describes how a child, trying to match the order of a series of doll's clothes on a line with a given series, may at a certain stage of development get a number of pairwise adjacencies right, but be unable to reproduce the whole order in the large correctly, because perceiving one kind of order may destroy the perception of a different kind of order.

At the stage in which objects are essentially only unconnected action patterns, the child has no notion of conserved objects having a number of properties. In fact, when he first begins to look for vanished objects at all, and finds one, the next time he looks he will look where he found it the first time, even though you have placed it behind a pillow in a different location before his very eyes. Piaget is very interested in the development of connections between the schemata, and this involves means of conservation of schemata and relations. A basic ingredient in conservation is the employment of actions for the prolongation of interesting events. Some are built in--e.g., eye movement reflexes tend to follow a moving object and to overshoot where it was last seen, and this happens to be fortunately preadapted to become a useful prediction device in an environment in which things tend to move in straight lines or smooth curves, and thus it prolongs the sight of the object. Later, these eye movements become connected with hand movements, and we gradually attain the conservation of an object which is the intersection of a number of schemata.

The child sucks the nipple, then his tongue, then his fingers. The hand motion isn't coordinated with the mouth, although the mouth tries to maintain contact (prolongation of a good thing--conservation). When he happens to get his thumb in his mouth it stays for a while. (Arm motions are now used in conservation--but only while the hand is sucked.)

First the child looks to look before looking to see. Similarly, there is empty grasping to begin with. These activities are just what conventional machine design might consider "inefficient," but they are the reason why the child has versatility and learning ability in contrast to conventional machines. There is "generalizing assimilation"--he looks at more and more things. Assimilation expands and incorporates more things--he sucks everything, coordinates sucking and hand movements. When he grasps he will grasp everything. Gradually visual schemata assimilate manual schemata--he will grasp what he sees. At first there is no coordination between vision and grasping, but if the hand comes by chance into the visual field it remains there (conservation and prolongation of actions again). The hand conserves what the eye looks at. Previously, tactile objects were "aliments" for manual schemata (aliments in the sense that the schema takes them in and grows on them), while now visual images are also aliments for manual schemata.

As in the example from the Bionics Symposium, pulsation of the hand during nursing is a kind of overflow of activation which leads to a general schema of taking in, which comes to include grasping, sucking, looking at, etc. This illustrates my factor group metaphor--one structure may be built up out of the ingredients of another structure. All sorts of actions (even personality traits) can contain the taking in schema. In a machine this would be necessary in order for the machine to perform acts, not just movements. (We can write with the pen held in our teeth and it comes out the same handwriting, though all that is in common between the muscle groups involved is this schema of a pattern, not of individual movements.) A child may purse his lips in accompaniment with any form of attentiveness, or open his mouth when trying to understand how to open a box. When the infant imitates you he will imitate according to these schemata--for instance you open your eyes and the child may open his mouth. Thus we can see how to trace these schemata, and be inspired to consider as essential in perception and in perceptual construction the interconnection of actions and perceptions through these schemata.

Piaget describes a form of recognition which appears to be intermediate between immediate automatic response to a stimulus and formation of a true concept that is mobile (usable in various contexts and combinations) and detached from immediate overt response to a stimulus. This form of recognition, which makes use of the excitation of a response mode by a stimulus, may be illustrated by one of a number of examples given by Piaget. One of his children had gone through a period during infancy when he delighted in striking objects to make them swing. One day, however, the baby happened to be engaged in an effort to grasp, not strike, some object which was suspended over his cradle. In one of his efforts (for his coordination was not yet completely developed) he chanced to make the object swing. Immediately he stopped trying to grasp, paused, shook his leg, and then resumed his efforts at grasping the object. Let us analyze what this means. A younger infant, if he had any schema at all for responding to thereby prolonging a desirable stimulus, the shaking object, would immediately use this schema; for instance, he might hit the object. If, at the moment he was engaged in something else and not sufficiently motivated to respond by hitting the object, he would do nothing. An adult might, on the other hand, say to himself, "Aha, here is an object capable of being swung, which is a good thing, except that at this moment I would grasp it; but I shall keep in mind the fact that it is swingable and remember which motions I can make to set it swinging." The baby in the present example does something intermediate. His response is not purely conceptual as in the latter case, and it is not a full and immediate response (or no response) as in the former case. It is instead a form of motor recognition through activation of a motor schema. This type of recognition by detection of a motor response to the form of a stimulus--by seeing what the stimulus makes the system do--could be investigated in a mechanical system.

In his book, Play, Dreams, and Imitation, Piaget develops his concepts of symbolic thought as a kind of equilibrium involving assimilation and accommodation, with the polar forms of extreme emphasis on accommodation and extreme emphasis upon assimilation represented respectively by imitation and by play. I have already spoken about imitation, and I'll just point out that the child first imitates you only if you are doing something involving a schema which is at the moment already active in the child--another example of the shaping of activity out of what is already being done, rather than merely out of transformations of inputs. This does not just mean that any function of inputs is also a function of internal parameters, which is obviously known to any machine builder--it is really a reversal of emphasis from "stimulus ⟶ response" to "internal production ⟶ excited by environment ⟶ modified by environment ⟶ combined with other internal productions." In relation to play, I was struck, immediately upon beginning to read the book, of the great importance of play in the development of perception and thinking. Play is the extreme form of assimilation--all sorts of things, inappropriate from a "rational" standpoint, are assimilated to a schema--the child "makes believe" by behaving toward one object as if it were another. By actively trying out objects in connection with schemata, the child can find out what fits what schema, and which things have which aspects of other things. In another form of play he freely activates schemata without regard to their "usefulness" and thus becomes capable of controlling the details of the schemata. It is my growing conviction that a machine which can "really" perceive must not only build its percepts out of actions and out of its previously formed repertoire of schemata as said before, but must be continually active between the times when you are using the machine for some practical purpose trying out its schemata in all sorts of combinations.

In summary of the above, some of the basic ingredients for really perceptive machines may be (and I think they must be) the employment of action schemata in the construction of acts and momentary spaces, incorporation of new aliments for schemata, reproductive, generalizing, and recognitory assimilation (depending upon or leading to state of equilibrium or disequilibrium), conservation of "interesting" states, "playful" exercise of schemata in various combinations. Now these terms are all preliminary characterizations of numerous examples, and the elucidation of what they might mean in specific situations of interest to us is the necessary first step which I am trying to take. I'd like to show the kind of relevance these considerations might have for machine construction by giving a single example.

This example will concern the ability of a machine to perceive things in Euclidean space. This involves some representation of the connection between different points and of a certain kind of homogeneity, in addition to other things, and so involves more than simply labelling things with one or more numbers. For instance, someone in the 1959 Western Joint Computer Conference was talking about a digital computer program to do a task which happened to involve square arrays of numbers. At one step in the program, the machine didn't notice a particular kind of resemblance between certain sets of numbers which a person could spot right away. This was because spotting the equivalence entailed interchanging rows of the matrix, or in other words, doing something that was easy to state in terms of spatial properties of the matrix which we can see when we look at it, but which were hard to state for the machines--looking at elements which lie along some straight line would be a good example. The authors state that the trouble is that we know how two-dimensional Euclidean space is interconnected--the lay of the land, so to speak, while the machine sees only points. Hermann Weyl in <u>Space, Time, Matter</u> shows how the scheme of geometry is built up from the conception of equality combined with that of continuous connection. If the notion of equality based upon congruence, is limited to translations as the examples of congruent transformations, you get affine geometry. It is possible to start with an unconnected manifold (like that of the computer) and successively introduce an affine connection (which tells how vectors at one point are related to those at other points) and finally a metric. Piaget, incidentally, traces the development of spatial perception from a topological stage when such things as enclosure are noted, through a projective stage to a metric stage. Piaget, in The Child's Perception of Space (from which I gave one example of perceiving according to the excitation of schemata in my University of Illinois Symposium paper) has a multitude of examples of how perception is not a one-shot transformation of inputs (though these are of course involved) but a matter involving a number of actions on the part of the perceiver when he perceives by the use of schemata); but here I just want to discuss the connection between points of space. This paragraph has indicated the existence of the question as a practical problem in computer design, and has indicated some of the mathematical requirements which must be met. Now I can show how the kind of psychological thinking I have been presenting can say something about the problem.

William James, in Chap. XX of The Principles of Psychology considers the problem of perceiving such relations, and in particular, the question of how, given a sensitive surface stimulated at two points, one can be aware of the duality of the excited points and of the extensiveness of the unexcited surface between them. James' view is that the relation between two points is a sensation of the line joining them, and that such a sensation could arise from the spreading with decrement of excitation from any stimulated point. Positions on the retina, according to James, might be discriminated on the basis of similar spreading, together with their relation to the fovea, for there is a tendency for the eye to turn so that the thing looked at is projected upon the fovea. Now James' chapter certainly doesn't answer the questions entirely satisfactorily, but it does have some idea worth looking into, especially the idea about turning to a central point which affords the clearest vision. This automatic turning of the eye to the position in which the stimulus can best excite the schemata of vision is the same kind of process as the reflex to turn the head in the direction of the cheek that is touched, so that the stimulating object such as the nipple ends up in the mouth. This similarity is one reason why the same schema of taking in comes to permeate both sucking and looking and ultimately grasping and all forms of attentiveness.

Let us consider such a process in an attempt to get a machine to introduce into space enough structure to hang the axioms of geometry upon. Suppose we have a sensitive surface which moves in such a way that the point of stimulation approaches some distinguished point, and which can record the motions it has made to center the stimulation. It will be easier to talk about the equivalent situation in which a point moves upon a surface in a pattern controlled by the machine and tries to get back to some reference point on the surface. The example will apply more to machines than to infants, because it involves sophisticated logical operations upon the rather simple input information supplied to the machine.

Suppose that the point moves in steps, turning left or right through some small angle at each step, and that the sequence of lefts and rights is recorded on a tape. Suppose that the machine can compare sections of tape of some standard length or lengths and know whether two such sections are the same or different, and sort such lengths of tape into classes containing identical sequences, or else sequences which are similar with respect to satisfying some criterion. Next suppose that the point moves at random or in some other way determined by the construction of its control mechanism, and suppose that some of its left-right sequences are recorded on lengths of tape. After some of these tapes have accumulated, let one or another of them be played back from time to time to determine motions of the point. Now suppose that the machine can tell when the point coincides with the reference point, and that it starts out at the reference point. Suppose that sequences tend to be played several times consecutively but tend to get erased if the moving point doesn't return soon to the reference point. The reference point, like the nipple or fovea, "nourishes" the action--in this case, perhaps, simply by activating a relay which keeps a demagnetizer turned off for a certain interval of time. For this example, we may avoid complications by assuming erasure such that the only tapes which survive contain sequences which starting from the reference point R, return to R, or sequences which when repeated once again will return to R (such as sequences corresponding to half the perimeter of a regular polygon, or any other path which ends up in the same place as such a path and facing the same way--i.e., rotated 180°). More generally, all sequences may survive, but the effect of returning to the reference point so soon might place a distinguishing mark upon the tapes for the special sequence just mentioned.

I am now following precisely the argument of Jean Nicod in "Geometry and the Sensible World" in Foundations of Geometry and Induction, Humanities Press (1950) written in 1923 as a doctoral dissertation. He defines (for paths in general taken by a hypothetical organism that moved about and was aware of proprioceptive stimuli plus an external signal at one point) a closed path as one which passes through a sequence of points of the form R........R. A null class is the class of all paths which can be inserted in a closed sequence (i.e., a path which starts and ends at the same point). Path y is the inverse of path x in case x followed by y is null. Path b is equivalent to c in case there is an x to which they are both inverse. He calls the equivalence classes unities. An alternation is a unity which is its own inverse, and a double alternation, a sequence of two alternations (not necessarily the same). Null paths and alternations are the two special kinds of paths mentioned in the last paragraph. Then it is easy to see that a double alternation places an object attached to the moving point at a different point of space but facing the same way, for the object ends up in the place it would be if it performed two semicircular movements with radii not necessarily the same. It is thus a translation. By defining equality and difference of alternations, Nicod succeeds in defining congruences and parallel displacements in the plane. In order to deal with translations in any plane--i.e., in three dimensions, he has to define a homogeneous double-alternation as one whose two components each commute with two other alternations, and it turns out that homogeneous double alternations and differences between pairs of them offer in terms of what this machine can detect an interpretation of points and their congruence adequate for construction of geometry.

Suppose that the machine also works out geometry with reference to a new reference point R', such that path class d leads from R' to R. Then you get the same kind of geometry by using dRd as the reference signal instead of R. Moreover, d may more generally be a path class or other kind of operation not contained in the first system. Then no "sensation" in the second system will be the same as any in the first system, and yet the two will be isomorphic. Nicod continues by pointing out that such systems can be mapped onto each other in such a way that they can all be coordinated into a total space. The map from one system to a second is simply the application to all points of the first space of an operation (or path) which leads from some point of the first space to some point of the second. The mapping is thus not uniquely defined--in fact any single point of the first space can be mapped upon an arbitrary point of the second space by an appropriate mapping. This is a manifestation of the homogeneity of space which has to be learned in a piecemeal way by this machine, as by a child.

Thus we see that an example of an action to conserve an interesting state--being at point R--in this machine, as well as in an infant can lead to a form of spatial perception, and in fact unite different perceptual spaces. We also see an example where perceptual behavior depends upon actions performed by the machine--it has to try out all sorts of paths and modify the tapes, and collect them into equivalence classes. The connection of space--that which distinguishes space from a mere collection of points--is achieved through sensori-motor activity. Now in carrying out such activities, all the one-step recognitions and modifications of behavior--that is, recognitions and modifications, however complicated, which do not involve hierarchical organizations of schemata--would naturally be carried out by the best recognizing devices now being developed.

The present problem is to determine what forms of spatial perception may be built up from various kinds of sensation. To perceive something spatially means more than having a static image of it--there must be a representation of various groups of transformations which may be applied to the image. Some of these transformation groups define the order constituting the perceptual geometry. Nicod considers four distinct orders of geometry. Two of these have as elementary relations the relations of global resemblance (things judged entirely same or entirely different) and temporal succession. They are thus concerned with the recurrences of the contents of perception. The first orders the particulars of any external sense, and arrives at topological relations of the environment. The second, which was discussed above, orders sensations of movement and internal configuration (proprioception or kinesthetic sense) mingled with recurrences of an enternal signal of invariable form (e.g., a pulse or other effect whenever some external reference point is reached). This is sufficient to give all of Euclidean geometry to the machine's representation of its environment. These two models lead to quite different orders of perceived relations, and I believe it will be fruitful to consider the problem of what internally and externally based perceptions mean in a mechanical device or in the models from ethology and Piaget, in which behavior is generated from intrinsic modes excited by stimuli.

The next two orders of perceptual geometry involve a single sense, such as vision. But this time there are spatial displacements of the visual data, and the previous global resemblance of two particulars divides into local and qualitative similarities (two experiences resemble each other because they were in the same place, or because they share a common quality, to describe them in our terms, not the machine's). Nicod first studies directly perceivable relations of position (local quality). These supply the only perceptual geometry in which simply stated theorems of ordinary geometry have simple perceptual interpretations; in the other orders of space, the usual geometrical relations receive meanings derived from complicated combinations of relations. This is why attempts to divide percepts into units corresponding to what we ordinarily think of as geometric units have not in my opinion been as fruitful as might be desired. Geometry based on relations of position also supply the only translation of geometry which is contained in an instant, for no other perceived space is present all at once. However, there are severe limitations on the construction of such a space.

Aside from this last space, the richest geometry is based upon local similarity (but not the above-required direct perception of positional relations) and qualitative similarity. It turns out to require simpler temporal relations, but a more highly differentiated order of similarity than the first two geometries. It needs only the relation of simultaneity, not order of succession. The notion of <u>points of view</u> may be introduced, and Nicod succeeds in attaching to an object the class of views perceptible from that object even though the perceived universe is purely visual and immobile.

Subtle differences exist between a three-dimensional field of vision that perceives equality of distances between pairs of points, and a two-dimensional field that sees equality of angular divergences of pairs of points, the latter depending upon the point of view. These differences involve spaces of objects and spaces of views and whether the first may be localized in the space of the latter. Passing from three-dimensional vision of a space full of matter to two-dimensional vision of scattered matter leads to the elimination of original orders and their reappearance in new and more complex forms, as well as the birth of new perceptual characters.

Two orders which may be perceived relate (1) to the content of individual views, giving us points of view in Euclidean geometry; and (2) relations involving the succession of views as a whole (if they resemble globally they are regarded as the same view, and if they differ at all they are regarded as purely and simply different). (1) studies contents, not times of occurrence, while (2) connects occurrences. The coincidence of these two orders is an empirical fact which can be discovered by the machine noticing that views encountered in time between views A and B derive their content from a class of views which in the perceptual geometry possess formal properties of a continuous line from A to B. (In perceptual geometry, various entities like lines are defined as certain equivalence classes.) Other relations connect this space with the kinesthetic space. The order of views will not tell about kinesthetic sensations, but two views separated by the same kinesthetic sensations are separated by the same distance in the geometry of views, and views succeeding each other in perceived time are separated by sensations of movement forming a continuous line in kinesthetic geometry.

See Section 2 of this report, "Summary of Research," for a proposed generalization of these ideas to a space which is not the physical space in which the organism moves, but a multidimensional "phase space" representing the configuration of activities in all the neurones, muscles, etc.

5. From 6th Quarterly Progress Report, Office of Naval Research Information Systems Branch Contract No. Nonr 2121(17) NR o49-149 Feb. 28, 1961

In addition to material described in publications, the following studies have been made:

1. General vector space formalism

Why is this needed at all? For instance, why is it important specifically to let probabilities be proportional to squares, and to use other aspects of a formalism inspired by quantum mechanics? I have previously given some reasons, but I still want to know, just which details are essential, and what kind of behavior arises from what elements of the model.

One attempt to develop a similar formalism from the properties of observation is that of Bastin and Kilmister, in a series of papers in the Proc. Cambridge Phil. Soc. and elsewhere, who try to find a foundation for theoretical physics in these considerations. Their degrees of success in this aim is of no concern to me from my point of view, for if they start with a characterization of observation which is of interest in machine perception and validly derive a formal structure like the one I want to put into networks, then their work would be of use in building networks, whether or not it is of use in understanding atoms. Now in my opinion these papers contain a number of arbitrary statements and choices, and a number of ambiguities, but there are a few points which are really suggestive. In fact, I had read Bastin and Kilmister, then read The Nature of Intelligence by L. L. Thurstone, and then returned to Bastin and Kilmister and found that their model was intended to account for certain properties which were specifically discussed as fundamental in the book on intelligence. So the question is whether something useful for my purposes can be obtained from the papers. So far my study of this gave me some ideas which may be useful, but only a few frustrating ideas specific to the model.

(a) I was able to obtain a somewhat clearer idea about the meaning of probability amplitudes and how they differ from probability, by separating out to a certain degree the part of the formalism that is pure convention with no physical meaning, and the part that makes a difference. For instance, if there is just one frame of reference, you can talk about any perfectly ordinary situation involving probabilities (e.g., the probability of getting a straight flush, or the probability that the Democrats or Republicans will win) in terms of probability amplitudes, and represent the state of the card deal or national situation by a vector having probability amplitudes as components, without saying anything different about the situation. It's just a tricky way of writing the same thing. The new effects come into play only when there are chains of conditional probabilities.

(b) Why make the seemingly arbitrary choice of a unitary geometry using quadratic norms? (i.e., squares in particular for probabilities) One possible reason comes from a paper by Hugh Everett III (Revs. Mod. Physics 29 (1957): 454-462) which deals with a notion of "relative quantum states" of parts of systems with respect to other parts of the system. The author considers systems in which we take the boundary to include the observer, so that only unitary transformations in time take place, and not projections with the introduction of random phases associated with observation. But because the system including the observer is so complicated, you can only deal with an ensemble of possible interactions which may have occurred, and you have to assign a probability measure which for certain natural reasons turns out to be quadratic.

Another reason is suggested in the paper of Bastin and Kilmister dealing with quantum mechanics, but their construction of the model seems inadequate in a number of respects (and I spent a lot of time being frustrated in attempts to make a satisfactory model, for it is very tempting from the point of view of self-organizing machines). They arrive at some vectors, and they say that only the direction, not the length, counts. Therefore, they say, they will consider only transformations which preserve length (seemingly a non sequitur), and the natural length, or norm, is a quadratic norm based upon an inner product (which must be shown, not assumed as they do). I think it can be said that other transformations make perfect sense, but that everything that may be learned from them can be learned from examing transformations that are isometric on the unit sphere; but isometric on the unit sphere implies isometric everywhere, and this implies linearity, so we may consider matrices which transform between coordinate systems (as I have been doing). Now if we want a general sort of model which is adaptable to all sorts of transformations, we want the existence of enough isometries to transform any unit vector into any other unit vector. Now it is, I believe, an open question in mathematics whether the existence of that many isometries implies quadraticity of the norm so that the normed vector space is a Hilbert space, but the condition, satisfied by a quadratic norm, does succeed in excluding any reasonable norm you can think of, such as any LP norm, p from 0 to ∞, with the exception of $p = 2$, the quadratic norm, so the argument is undoubtedly valid for all norms or at least for any norm except some pathological ones that couldn't ever be realized physically.

In my WJCC paper I told how the requirement that the system be capable of having "sharp" states that didn't get mixed up with others despite the transformations, suggested the unitary space. If we do not require, as in the preceding paragraphs, that all lengths be preserved on the unit sphere, but only that sharp states exist, i.e., in terms of the model, only that the transformations preserve orthogonality (but no other angles), it still turns out that the transformation must be a diagonal matrix times a unitary matrix, and the diagonal matrix just means a change of units, and make no essential difference.

In the WJCC paper I said that when you take squared magnitudes to get probabilities you throw out phases, so that a state vector, as is well known in quantum mechanics, can have some extra information packed into it that doesn't show up in some particular coordinate system, but that encodes potentialities for behavior in other systems. If we ignore for the moment the fact that we specifically take squares, we may consider the more general facts that we deal with transformations among an additional set of entities related to but not equal to the probabilities, and that the transformation from these entities to the probabilities is many-one. The necessity for this is suggested by the arguments of the preceding paragraphs. The requirement of sharpness implies that transformations from one system to another be non-singular. Now suppose that the probabilities equaled the former entities (amplitudes) or depended upon them in a one-to-one fashion. Since we want a general purpose model relating all coordinate systems symmetrically (until specifics are put in) each probability is the same function of its corresponding amplitude. Then whenever all probabilities happened to be the same all the amplitudes would be the same, so that the amplitude matrix, in this or in a host of other situations, would be singular. Therefore, the amplitudes must not be related to the probabilities in any one-one way. This argument is not the same one which led physicists to use amplitudes, although it is involved in Landé's heuristic reconstruction of the physical theory. It is of interest in this connection to note that a number of mathematical learning theorists find it profitable to consider models separating learning from performance, with different laws of combination at the two levels. (See Bush and Estes, Studies in Mathematical Learning Theory and Luce, Individual Choice Behavior.) In my model there is an analogy between the amplitude and the learning theory learning, or habit strength; the probability and the learning theory performance probability; and the strength of the random excitation used in eliciting components and the learning theory drive level which elicits responses.

2. Ideas from ethology, the study of the structure of animal instinctive behavior.

We all know that animals carry out remarkable acts of recognition and highly integrated patterns of instinctive behavior. What are the lawful relations constituting this integration, and what kind of physiological processes are capable of their physical representation? To pick an example arbitrarily, which involves learning to recognize patterns, various animals have predispositions to learn particular things. For instance, a toad, who is much stupider than a rat at most learning tasks, can learn to recognize and avoid eating an unpalatable insect after one or two trials, distinguishing it thereafter from all other insects. Since unpalatable objects such as some insects, rocks, etc., do not come in any one particular shape, the basis of this remarkable ability cannot be a simple filter preset to certain characteristics. The toad must learn to recognize any of a variety of objects; yet he does not recognize so well in any other context, only in that of eating things. Similarly, many European songbirds learn in one trial not to eat wasps, which have a repellent taste to them and may sting. Digger wasps have an amazing learning capacity for finding and recognizing the landmarks indicating their nests, but show no signs of conditioning in their hunting and food-seeking behavior; and they have an innate "program" of releasing mechanisms for "sub-routines" directing the chain of prey-hunting activities to the hive bee alone, among hundreds of other insect species. One value in understanding such phenomena is that we might be able to learn the characteristics of what must be learned and what has to be built in when you are trying to be especially good at a particular learning task.

The preceding paragraph was intended just to give a bit of the motivation for studying such things, and not to indicate the particular questions which I have been studying. The patterns of recognition and integration of behavior which were of most interest to me in connection with the design of a machine involve production of and response to a sequence of signals which occur in the interaction of pairs of animals in fighting and courtship ceremonies. I didn't use these examples in the preceding paragraph because I wasn't sure that in a short paragraph I could make you believe that they had anything to do with the problems of machine recognition--but they do. And the motivation doesn't really involve just the overall idea of that paragraph that animals can recognize things and wouldn't it be nice to know how, but it really involves a number of individual mechanisms that have been discovered by the ethologists, which are beyond the scope of this progress report to discuss. In general, however, most recognitions and conditional transfers are improved by learning but are constituted in the first place of specific sign stimuli which release actions consisting of two components, (a) a fixed action pattern which, once triggered, proceeds to the end even if the stimulus is taken away, and (b) an orientation component dependent upon the continued presence of the stimulus. The animal's experience is built out of these modes of activity of the animal, as pointed out in my remarks from the Bionics Symposium, and some of the most complex acts of recognition and behavior patterns may be accounted for on this basis. Also, in my WJCC paper and in the <u>Univ. of Ill. Symp. on Principles of Self-Organization</u> I pointed out certain analogies of my model with the way the patterns of instinctive behavior are organized.

As pointed out in the <u>Ill. Symp.</u>, review of the literature of developmental and comparative psychology reveals the impressive extent to which the animal reacts to his surroundings by fitting to the surroundings certain patterns of activity which it is inherently able to perform, or which it has developed through previous activity. It does not seem to start with a homogeneous network upon which it depicts experience; rather, experience seems to excite certain patterns inherent in the network, which are then reshaped to fit better.

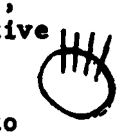
As an example, the anemone has a complicated pattern of activity which may be changed by stimulation. A brief exposure to food leads to a prolonged series of changes associated with feeding, digestion, and elimination. But it is reported that the animal will sometimes run through the whole sequence without any evident external stimulation.

Thus the complex feeding behavior of the anemone appears to result from the triggering of a pattern which it can perform independently of any food, but which becomes useful in the right situations. According to this interpretation, stimuli may best be regarded as causing the anemone to shift from one of its intrinsic patterns to another.

The work of Lorenz, Tinbergen, and others has shown that instinctive behavior in general appears to depend upon the excitation of analogous innate mechanisms by specific "releasing" stimuli; and the role of learning in this aspect of instinctive behavior may be to make fine adjustments in the set of releasers so as to enable the animal better to meet the complexities of its environment.

Another example of the active role of the nervous system in constructing the units of perception prior to their fitting and adjustment to the environment comes from the way children perceive and represent shapes by drawing them. For instance, a young child may respond to visual patterns by "drawing" a circle, square, and cross all as unintelligible scribbles. Later, the circle and square come to be represented by roundish scrawls, while the cross is represented by slashing scribbles. Next the swirling that represents circle and square becomes sharpened into a single circular outline for each. This same outline serves to represent any closed figure. At this stage, the child is good at representing closure or openness of curves, and relations of contiguity. At a still later age, the circle is still represented by a circular outline, while the square is often represented by a circular outline with four short lines intersecting the outline at one place or another.

This behavior is what one might expect if the child's perceptual and motor networks have naturally occurring swirling and slashing modes all over them. These might be built in from the start, or they might arise from motor activity and from the passage of light over the retina during natural movements of the child. Then the circle and square are more likely to excite the swirling modes than the slashing modes. When the child is a little older he fits these modes to the pattern by suppressing most of the overlaid swirls or slashes. When he has developed still further, and, let us say, can perceive the finer aspects of things, he is sensitive enough for the square (which has straight sides and relatively sharp corners) to excite a second component, the slashes, so he puts them in the picture. He puts them close to the swirl because he is sensitive to contiguity (a natural property of neural nets), but he does not integrate both modes into the same structure (closed <u>and</u> sharp) as he must if he is accurately to represent the square.



This independence of modes is also seen in the independence of innate releasing stimuli in the theory of instinct. These specific characteristics, which when perceived set off a behavior pattern, are generally completely independently acting stimuli which may easily be described in words as independent characteristics of the environment.

We may say that it appears that the child or animal has certain representational or motor patterns which are used, if they happen to be useful, in reacting to the environment. The child seems to draw something by letting it excite the closest inherent patterns which he happens to possess. Then he begins to sharpen the result to make it fit better. Ideas like these suggest that a perceptual machine might work better if it followed a stage of processing that contains such excitable structures. Then a circle would be perceived anywhere in the network because anywhere in the network such a swirling mode could be excited.

In connection with these ideas, and in addition to the material contained in publications, I have done the following during the period covered by this report:

(a) I have studied some of those aspects of ethology which I felt might be relevant to problems of perception and knowledge in animal, man, and machine. To this end I also attended two courses given by Dr. Irenäus Eibl-Eibesfeldt of the Max Planck Institute, an associate of Konrad Lorenz, who has carried out ethological experimentation and observation in Germany, the Indian Ocean, and the Galápagos, the last under the auspices of UNESCO, and who was in Chicago this winter. Valid interpretations of the various components of the behavior can be made only through comparative study, tracing them through various species, and tracing the derivation of components from other components revealed in the total behavior patterns of the animals. This material is not as well known to people in our field as it should be because it has been carried out for the most part by European zoologists, and has largely been ignored by American psychologists who have other interests, so I think it is especially important to obtain an over-all view of the field, because there is no other way to know in the future whether the information will turn out to be relevant. But more immediately, most of the ideas I have been saying all along about my model for perception have been influenced to a great degree by what I knew about ethology. The next paragraph tells about a specific type of mechanism.

(b) I don't want to go into the details here, but it appears that courtship behavior in many birds and fishes depends upon the relative amounts of three conflicting drives, to attack, to escape, and to be near or mate. The state of the animal in this three-dimensional space determines the posture he assumes and his motion relative to the other animal. Without going into the reasons for this or the ways in which it occurs, I will only point out that the orderly sequence of postures and motions serves as a set of symbols making possible recognition of the proper partner and the proper circumstances, and releasing the proper conditional transfers in the "program" leading to successful consummation of the mating behavior. Similar processes may be noted in other kinds of behavior.

The way in which these modes of motor response mediate recognition is reminiscent of the example of motor recognition cited in the 5th Quarterly Progress Report, above. This was an example of behavior intermediate between direct response and conceptual thought, in which the child recognized the presence of a shakeable object by shaking his own foot. The question arises: can a self-organizing system develop stable modes of response that can be used as elements out of which to build recognition and motor behavior, as in instinctive behavior, Piaget, and perhaps the computer-controlled mechanical arm described by Shannon in Current Research and Development in Scientific Documentation, No. 7. I began to study a special case of this question, and I shall say a few words about this.

Suppose we forget for the moment about the complications of the nervous systems of birds and assume that they are simply linear systems. Then the input to A's motor system is the output of this system as transformed successively by the receptors of B, B's motor system, and A's receptors. Lump all these transformations together and we have a feedback system in which the input is some linear transformation of the input in the steady state, let us say, $x = Rx$, where x is the input vector (three-dimensional in this case) and R is a matrix. Before the steady state, if any, is reached, we might have something like $dx/dt = Rx - x$.

The same equations, or more complicated versions, might describe the behavior of a mechanical arm in hand. Here x might denote the set of inputs to the device controlling the position of the hand, and proprioceptors might feed back to these inputs information about the position of the hand; or visual receptors might send back information about the position of some object manipulated by the hand.

In general, however, there will be no steady state, because in general there will be no solutions of the equation $Rx = x$. The closest we can come is $Rx = \lambda x$, where λ is any one of the eigenvalues of R. We don't know in advance what λ will be, so we can't set up a system described by the last equation. However, it is very natural to assume that the state of the system is determined by the relative strengths of the inputs. For instance, the direction of movement might be determined in such a way, with the intensity of the movement being determined by the absolute magnitudes. Then the state is effectively determined by $x/|x|$, and we have the equation $dx/dt = Rx/|x| - x$. This nonlinear vector equation may have steady state solutions of the form $x = \lambda u$, where u is an eigenvector of unit length and λ is the corresponding eigenvalue. Thus, although the absolute value of the vector doesn't make any difference to the network determining the motions, the steady state solutions come out having definite lengths as well as directions. In this system the output is divided by its absolute value and fed back into the input. Some systems would automatically behave in this way without having a special device which actually performs a division.

I have made some preliminary studies of such systems to see whether they would arrive at stable configurations and patterns of motion which could be used as units out of which to build recognitions and actions. So far it doesn't seem worthwhile to try to solve the highly nonlinear equations analytically. Remember, even the simple equation above means a set of equations $dx_i/dt = \Sigma R_{ij}x_j/(\Sigma x_k^2)^{\frac{1}{2}} - x_i$ But at least for symmetric R, I can show by means of the variational properties of the eigenvalues, that solutions are stable in the vicinity of some of the vectors λu, and I can show some of the possible modes of approach to the steady state by graphical means.

This is all in a very preliminary stage, but the reason I think I may get some good ideas from pursuing it is the following sort of thing. If a point in the phase space of x happens to be placed upon an eigenvector, it stays on that eigenvector; and if it is placed in a plane determined by two eigenvectors it stays in this plane. It might go into another plane determined by two other eigenvectors only by moving close to an "edge" corresponding to another eigenvector, and if it is close enough to the origin (i.e., of small abolute value) perturbations may send it into another plane. That is, once the point gets into certain subspaces -- corresponding to whole classes of motions -- it will tend to linger in the subspaces. Now if you will refer to the <u>5th Quarterly Progress Report</u>, you will see that a basic mechanism found by Piaget to contribute to the construction of spatial experience is that of conservation -- the prolongation of desirable situations. Of course, everybody talks about conservation of things that are successful, but Piaget discusses the actual ways in which these processes lead to spatial perception, rather than merely pointing out that they would be beneficial, or else being specific but on a level which is not really perceptual. Piaget is both specific and concerned with things involved in the development of spatial <u>experience</u>. Consequently, it is valuable to expend some effort just trying to see various ways in which his mechanisms could be active in simple models of various kinds, and gradually find out which features I want, rather than working out one of the models completely before knowing what I'd do with the model if I knew all its details.

To investigate the formation of stable units in a system such as I have been describing I am going to start by including some of the features Piaget has discussed in arbitrarily simplified versions. For instance, in an ecology, to use Brunswik's term, in which objects tend to move in smooth curves, some form of inertia tends to be a good predictive device that prolongs sensory contact with the object and thus is a step toward breaking up the visual field into objects -- we want a machine that sees <u>things</u> and not all the outlines produced by random combinations of objects, points, and spaces between them. Considering once again the frequently mentioned conservation of things that are successful, Piaget shows just where and how this mechanism operates in the development of space and object concepts. For instance (see notes on <u>Construction of Reality in the Child</u>, below, Stage 3, Section 1), at one stage the infant searches for objects taken from him only if he was already in contact with them and his searching movements are prolongations of motions he was already making. The object does not yet have an autonomous existence for him; it is for him only a combination of his own actions. It is easy to think up versions of this that could be put into a mechanical arm.

In this section we have seen that some ideas originally inspired by ethological studies suggested mechanisms to look for in space perception.

6. SOME INGREDIENTS OF PERCEPTION

(What follows consists of some notes on Piaget's <u>The Construction of Reality in the Child</u>. These notes are not intended to summarize the book. They are not complete, and they do not include the examples which give content to the ideas. The notes do not describe the six stages, as does the book, but are intended only to list a number of interesting ingredients of developing experience. One wants to know in detail just how such a set of ingredients can generate experience.)

CHAPTER I. DEVELOPMENT OF OBJECT CONCEPT

Stages 1, 2: No special behavior related to vanished objects p.4.
Stage 3: Beginning of permanence extending the movements of accommodation, p.13.
Stage 4: Active search for the vanished object, but without taking account of the sequences of visible displacement, p.44.
Stage 5: Child takes account of the sequential displacements of the object, p.66.
Stage 6: The representation of invisible displacements, p.79.

<u>Stages 1 and 2: (p.8)</u>

Intersensory coordinations (sucking-prehension, prehension-sight, etc.) contribute to arousing anticipations which are assurances of the solidity and coherence of the external world. Though they contribute to solidifying the universe by organizing actions, they do not suffice to render that universe external to those actions, pp. 8-9.

Sensorimotor accommodations often lead not only to <u>anticipations</u> concerning perception (such as above-mentioned coordinations), but also to extensions of the action related to the image perceived, even after the image has disappeared. E.g., visual accommodations -- when the child can follow with his eyes an image which is being displaced, and when he can extend that eye movement by appropriate head and torso movement, he reveals behavior tantamount to <u>search</u>, p.9. Contact with lips gives rise to slightly systematic pursuit once they have disappeared. (Not like true search, which comes much later.) Continues to watch point where person left visual field, or point where frequently enters visual field-- he "expects" resumption of interrupted feelings, i.e., performs "searching" or else action appropriate to its presence; in reality just a continuation of what he was doing. Same with prehension, p.10. (Obs. 5 -- looks at one picture, then at second, then back to first -- example is equivalent, in realm of primary circular reactions, of the deferred reactions to be analyzed in Stage 2.)

These behavior patterns aren't true search. True search is active and causes the intervention of movements which do not solely extend the interrupted action. At present, expected object is still related to the action itself (i.e., not related to permanent object with external existence). He only looks where it has vanished -- can't search, remove obstacles, change position, etc. What search there is, merely reproduces earlier act of accommodation. In the case of sucking, it is a reflex mechanism which allows the child to grope until he encounters the objective. The objective is in the direct extension of the act.

So in first two stages, no special behavior related to vanished objects - they remain at his disposal, ready to return if he cries or looks at the place where he saw them last. Initial search not an effort to understand displacements, but only extension or repetition of most recent acts of accommodation.

STAGE 3: BEGINNING OF PERMANENCE EXTENDING THE MOVEMENTS OF ACCOMMODATION

Behavior patterns between beginnings of prehension for things seen and beginnings of active search for vanished objects. Reference: O.I., Chap. II, Sec. 4. Begins to grasp what he sees, brings touched objects before eyes (3-6 months). No active search and removal of covering objects until 9 or 10 months, p. 13. During interim acquires series of intermediate behavior patterns:

1. Visual accommodation to rapid movements. 14ff
2. Interrupted prehension (follows objects which have escaped his grasp). 21ff
3. Deferred circular reaction (spontaneously returns later to interrupted action. 24ff
4. Reconstruction of an invisible whole from a visible fraction. 27ff
5. Removal of obstacles preventing perception. 32ff

1. Visual accommodation to rapid movements:

Makes possible the anticipation of future positions of the object and consequently endows it with a certain permanence. While this permanence is still rooted in the act of accommodation itself, there is progress in sense that the anticipated position of the object is a new position and not one observed a moment earlier to which the eyes merely return. Two instances of special importance: (a) reaction to movement of bodies which disappear from visual field after having induced a lateral turn of the head; (b) reaction to falling movements.

Both these behavior patterns seem to have developed under the influence of prehension. For instance (Obs. 7) visual search for fallen object is more intense if object has been previously touched or grasped.

If experimenter (E) drops object, the subject (S) may look around E's hand, or follow trajectory, or else look directly at floor without having had time to follow the trajectory and having seen only point of departure and E's empty hands. But with another object the reaction may be completely negative, S examining E's empty hands in astonishment. Hence the object concept doesn't yet exist; in the case of the successful reactions it is simply the movement of accommodation which continues, and when the object is too small for the eyes to follow at the point of departure nothing happens.

(Obs. 11a gives another deferred reaction like Obs. 5--S looks at E_1, then watches E_2 as latter leaves, then turns head back to look directly at E_1--"knew E_1 was there" even though hadn't looked at him for a few moments.)

Thus the beginnings of permanence attributed to images perceived arise from the child's action in movements of accommodation. Extension of Stage 2, but with progress: child no longer seeks object only where he has recently seen it, but hunts for it in a new place. He anticipates the perception of successive positions of the moving object and in a sense makes allowance for its displacements. But precisely because this beginning of permanence is only an extension of the action in progress, it could only be very limited. The child cannot conceive of just any displacements or just any objective performance. He is limited to pursuing, more or less correctly, with his eyes or with his hand the trajectory delineated by the movements of accommodation peculiar to the immediately preceding perception, and it is only insofar that he continues these actions after the objects have left that he is able to endow them with a certain permanence.

Search takes place more often when child himself has let object drop. The displacement attributed to the object depends essentially upon the child's action (movements of accommodation which are extended by looking) and permanence remains related to that action. Followed object not regarded as being in a space independent of himself; child not aware he is moving to follow the object. If object is not found within the exact extension of the movement of accommodation, the child will give up trying to find it. Object's movement is one with sensorimotor impressions which accompany his movements; when he loses sight of object the only procedures suitable for finding it again consists either in extending movements which have already been delineated or in returning to the point of departure. There is still no particular behavior pattern related to vanished objects.

2. Interrupted prehension: 21

Permanence of tactile object is still only an extension of accommodation movements, but henceforth the child will try to grasp the lost object in new positions and no longer only in the same place. Prehension becomes systematic and of surpassing interest between 4-6 months, and child learns to follow with his hand objects which escape him even when he does not see them. Beginning of permanence.

Take object away from child's grasp, and child will search near place where the object departed. But this search is dependent upon prehension. If hide object, before her eyes, child does not look for it. Replace in child's hand and take it again -- child searches. Obs. 14: E places on S's lap an object S has just held in her hand. Just as she is about to grasp it again, E puts his hand between S's eyes and the eraser; she immediately gives up. In ten repetitions of the experiment, every time S is touching the object with her finger at the moment when E cuts off her view of it she continues her search to the point of complete success (without looking at the object); but if no tactile contact has been established before the child ceases to see the object, she withdraws her hand.

Obs. 15: S grasps cloth covering bassinet, lets go, with hand tightly closed, brings closed hand before eyes, and opens it cautiously. Looks at fingers attentively, and repeats whole sequence more than ten times. Thus sufficient to have touched object, "believing it has been grasped," to act as though object is still in hand, although no longer felt. Degree of tactile permanence. Obs. 16: S loses object which has just been grasped. Brings hand to eyes and looks at it with surprised, disappointed look; then he swings his hand although it is empty, then looks at it once more. Object as extension of action.

In contrast to searching in Stage 4, the above is still only a question of permanence merely extending earlier accommodation movements, and not of a special search for the vanished object. Child merely reproduces the gesture of grasping which he made shortly before. Child is content merely to stretch out his arm; the least obstacle makes him give up. But later search procedures will develop out of the present patterns.

3. Deferred circular reactions: 24

So far object is that which is at the disposal of certain actions. The activity at this level consists mainly in primary and secondary circular reactions, and not yet in tertiary reactions. That is, he mainly reproduces interesting results and tries only a little to study new things for their own sake, to experiment.

Now we consider acts in the course of which circular reaction is interrupted by circumstances and resumes shortly after without any external stimulus. Additional permanence of object. Still only global, practical permanence, with nothing implying that objects removed from present context will remain identical to themselves. It is not the object which is permanent, but the act itself.

4. Reconstruction of an invisible whole from a visible fraction: 27
One might expect child to try to see familiar object as a whole after he has caught sight of part of it right from earliest stages, but Piaget has not definitely observed this until after prehension has been acquired. Part of object (or sufficiently large part) is enough to make child look for rest of it or produce his usual reaction to the whole object. If hide the object completely, the child stops reacting to it. Thus Piaget considers the act of reconstructing a totality from a visible fraction of the thing to be psychologically simpler than the act of searching for a vanished object.

If bottle disappears from visual field -- same as if ceased to exist. S cries from impatience when he sees his bottle. Hide it, and he stops crying. After several repetitions, he is enraged -- this shows he believes he can count on the object, that it is at the disposal of his desires.

No matter what part of bottle he sees, it is recognized, but if he can't see the nipple it is as though the bottle is incomplete, though he expects the nipple to appear sooner or later. (See Chap. II on Space.) Call this virtual totality: bottle is a whole, but its various elements are conceived as being at his disposal, and not organized in space.

5. Removal of obstacles preventing perception: 32
Various stages, leading up to Stage 4. At first, only when object is partly in view -- reconstructs totality as a function of the immediate action (stage 4 -- must momentarily give up his attempt at direct prehension of the object in order to raise a screen recognized as such). Present stage no more than he does to extricate toy from cover clumsily grasped along with it. If completely hidden stops searching or looks around hand which hid object.

Examples in which E places doll under covers; then he taps, making doll rattle. S taps but doesn't raise cover. E exposes tip of doll; S uncovers. E replaces cover; S taps on it but doesn't lift cover. At this stage, so long as search for vanished object merely extends accommodation movements in progress, child reacts to disappearance. But as soon as more must be done -- interrupting the movements in order to raise a screen conceived as such, the child abandons active search, even when he hears the object under the cloth. The permanence is still that of the action in progress; all that is assumed is that in continuing to turn head or lower hand he will regain an impression which has vanished. Child will frequently return to the initial position at beginning of experiment.
In Stage 3 we see beginnings of permanence of visual images and tactile objects, but the two are not yet merged. This coordination will occur in Stage 4.

STAGE 4: ACTIVE SEARCH FOR THE VANISHED OBJECT BUT WITHOUT TAKING ACCOUNT OF THE SEQUENCE OF VISIBLE DISPLACEMENTS

The child is no longer content to search for the vanished object when it is found in the extension of accommodation movements; henceforth he searches for it even outside the perceptual field, that is, behind screens. This discovery rises from the fact that the child begins to study displacements of objects (by grasping, shaking, swinging, hiding and finding them, etc.), and thus begins to coordinate visual permanence and tactile permanence. But object still remains intermediate between thing at disposal of actions and object proper, in that child acts as though the place where the object was found the first time remains where he will find it when he wants to do so, even though he has just seen the object transferred to a new place. (And in Obs.41, even though screen at 2nd position is lifted to show child that object is still there.)

Finding object behind screen occurs between 8-10 months. Many examples are described of transitional stages.

In next few weeks he looks in second position, but if he does not find the object immediately, or it has been moved to a third position, he goes back to the first position. Examples of transition to fifth stage. Child expects another person to be in his accustomed room, even though he has just been seen elsewhere. This is one of series of residual reactions analogous to Stage 4 which may reappear much later (e.g., at 2 years if the problem is more difficult).

STAGE 5: THE CHILD TAKES ACCOUNT OF THE SEQUENTIAL DISPLACEMENTS OF THE OBJECT 66

From the end of the first year of life until toward the middle of the second the child progressively acquires spatial relations, and he no longer looks for the object in a special position, but rather in the position resulting from the last visible displacement. But if one interposes the simplest possible of invisible displacements, the phenomena of the preceding stage reappear. E.g., if object in a lidless box is placed under screen, and the box is brought out empty, the child can not understand that the object is still under the screen. Moreover, the child may look in the first place even though he has seen the object in a new place. This exemplifies what Piaget calls the "law of temporal displacements" -- when an operation passes over to a new plane of action (in this case, that of representation of invisible movements) it has to be relearned (in this case he relapses into the same difficulties that were overcome when only visible displacements were involved).

STAGE 6: THE REPRESENTATION OF INVISIBLE DISPLACEMENTS 79

In general terms, the child has become capable of directing his search by means of mental representation. Sometimes he takes note of the invisible displacements of the object and shows himself able to deduce them as well as to perceive them; sometimes through thought, he masters a series of incasements too complex not to give rise to a true awareness of relationships. In some of the experiments, when the interference of memories began to make the task difficult, the child retraced the order E followed in order to recall which was the last screen. This presupposes representation of invisible displacements, and the object begins to attain permanence, since the law of its displacements is entirely dissociated from the action itself. When the object is placed in a box, and then the box is placed behind a screen we note additional complexity: the box becomes an object for search, while remaining a screen.

The object is no longer merely the extension of various accommodations (Stages I - IV), nor a body whose movements have become independent solely to the extent to which they have been perceived (V); it is freed from perception and follows autonomous laws of displacement. Searching before this stage has merely utilized a system of signs linked with the action; searching for an object S has just seen placed behind a screen does not mean that S "imagines" the object under the screen, but simply that he has seen the screen at the moment that the object was covered, and he therefore interprets the screen as a sign of the actual presence of the object. It is different to imagine the object when there is nothing in sight to attest to its hidden existence. Permanence has been extended to regions of the world which are dissociated from action and perception. A final consequence is that the child's own body is regarded as an object. Thanks to imitation, the child sees his body as an object by analogy with that of another person. He reverses his initial universe, centered on his actions, by transforming it into a universe of coordinated objects including the body itself.

DISCUSSION OF OBJECT CONCEPT:

Compare the child's objectification with that of scientific thought. In the sciences, something is objective if (1) it permits anticipation, (2) it lends itself to confirming experiments, and (3) it is connected in an intelligible way with the totality of a spatio-temporal and causal system. The child uses these same methods in forming an objective world: (1) At first the object is only the extension of accommodation movements (anticipation). (2) Then it is the point of intersection, i.e., of reciprocal assimilation of multiple schemata which manifest the different modalities of the action (concordance of the experiments). (3) Finally the object is included in an intelligible spatio-temporal and causal world.

The first contact with the environment, i.e., taking possession of things through reflex assimilation, does not imply awareness of the object as such. Although there is a capacity for repetition, generalization, and recognition nothing as yet forces the child to dissociate the action itself from its point of application. So long as his action succeeds, his objective is the same thing as his awareness of desire, effort, or success. The question of the object's independence and permanence arises only when the child perceives the dissappearance of desired objects and searches for them actively. During Stages I and II, the infant notices the disappearance of objects, and shows his emotions when the breast or the mother disappears. But his only positive reaction for finding lost objects consists in reproducing the latest accommodation movements he has made. The object is still only the extension of the action. The effort of accommodation arising at the moment of the object's disappearance at most foretells the need for conservation which will subsequently constitute the object. This elementary permanence is accentuated when in Stage III, the child no longer limits himself to searching for the object only where it has just disappeared, but extends the accommodation movement in the direction it followed up to then (reaction to the fall, etc.). The act of losing contact with the object momentarily to find it in a new position is progress in conferring autonomy to the object. But as long as the search consists merely in extending accommodation movements already made in the presence of the object, there is not yet intrinsic permanence.

On the other hand, progress comes with the coordination of multiple primary schemata. Some examples of this are the behavior patterns of "deferred circular reaction," of search for the whole when only a part has been seen, and the suppression of obstacles preventing perception (end of Stage III). In these the child combines visual and tactile searching. But real permanence begins only with the search for a vanished object in a comprehensible spatio-temporal universe. The three steps of this search characterize the last three stages: simple search without taking account of objective displacement groups, then search based upon the group of perceived displacements, and finally search involving representation of displacements not perceived. How can the child elaborate such relations and construct permanent objects, given the moving images of immediate perception? At first, active search merely extends the behavior patterns of Stages I-III. The child begins to pursue invisible objects only after he has made the movement of grasping when they are in sight. But even when searching takes place independently of this condition, the object is only sought where it was found the first time. Therefore it still depends on the action; it is not differentiated from the whole situation in which it gave rise to a successful search. The only progress is pursuing the object behind a screen, and no longer only when it is partly visible. Object gradually becomes detached from action. And the action ceases to be the source of the external world and becomes merely a factor among other factors. And this is the construction of a spatio-temporal network, and a system consisting of substances and of cause-effect relations. Hence the construction of the object is inseparable from that of space and time and causality.

How does the child come to search for the object, taking account of its sequential displacements? It is neither a priori deduction, nor purely empirical learning, but is a constructive deduction. That it is not simple deduction may be seen from the gropings needed to learn the relationships of displacements. These show the need for active experience in order to build up sequential displacements. But the gropings are not purely empirical; the experiments are directed. In finding the object the child organizes his schemata rather than submitting passively to the pressure of events. It appears that the permanence of the object stems from the constructive deduction which from the fourth stage is constituted by reciprocal assimilation of the secondary schemata, i.e., the coordination of schemata which have become mobile. Until then the object merely extends the activity itself. The coordination of the primary schemata, such as sight and prehension, results in some externalization of objects, but so long as the secondary schemata remain global instead of being dissociated, the better to unite, there is no real permanence. But from the fourth stage onward, the secondary schemata become mobile through a reciprocal assimilation which permits them to combine among themselves in different ways, and this leads to a spatio-temporal world of objects endowed with causality.

According to Origins of Intelligence in Children, Chap. IV, Sec. 1-3, the mobile schemata allow the construction of physical connections and thus of objects as such. Thus the union of the schemata of prehension with those of striking, which explains the behavior pattern of removing obstacles, permits the child to construct the relations "above," "below,""hidden behind," etc. Above all, the combinations of mobile schemata make possible a better accommodation of behavior to the specific characteristics of objects. The fact that the schemata can henceforth adjust themselves to each other leads the child to observe the detail of objects much more closely when his action bears upon them than when the objects are absorbed in the acts as a whole and remain undifferentiated. Thus Stage IV shows exploration of new objects, and Stage V shows tertiary circular reactions, that is, experiments in order to see. This process of combination of accommodation with reciprocal assimilation of the schemata explains the discovery of the object's real permanence. This discovery presupposes experience, for the failure of his initial search teaches him the object is where it was last hidden; and deduction, since without the reciprocal assimilation of schemata the child could not assume the permanence of objects and their existence behind the screen. Finally, in the sixth stage we find combinations of mental representations.

CHAPTER II. THE SPATIAL FIELD AND THE ELABORATION OF GROUPS OF DISPLACEMENTS

Spatial organization is correlated with the object concept. At first the object does not exist apart from action; it gradually becomes a permanent autonomous substance, and the subject becomes an object among other objects. Similarly, there are at first only practical spaces of various activities; space gradually becomes a framework of a universe in which all displacements are located, including those of the subject himself. At first there are only pupillary and palpebral reflexes to light. For space to be perceived as a container for objects, they must be placed in a framework of relations. Spatial construction may thus be viewed in terms of the six stages in the development of the object concept:

Stages I, II (no behavior pattern relative to vanished objects). Space consists of heterogeneous and purely practical groups (each perceptual bundle constitutes a space). Operations and their inverses exist, but the child does not perceive them or become aware of the motor operations which constitute them.

Stage III (the beginning of permanence extending accommodation movements). Spatial groups intercoordinate and become subjective. They intercoordinate under the influence of prehension (which connects a visual space to tactile and gustatory space) at precisely the time when prehension guarantees the object a beginning of permanence. By manipulating things the child becomes capable of imparting systematic movements to them and thus of perceiving groups in the universe itself. But these groups remain dependent upon sensory appearance and the child's own perspective, and since they are dependent upon the subject's action, we call them subjective. They are groups which connect a subject who does not know himself with a semi-permanent object, and not groups uniting objects as such with each other.

Stage IV (active search for the vanished object, but in a special position and without taking account of its sequential displacements). The child becomes capable of hiding and finding, and thus begins to form objective groups. He does not yet take account of sequential displacements, and Piaget calls this the stage of the "group of simply reversible operations."

Stage V (permanence of the object throughout its displacements). Advent of the objective group.

Stage VI (representation of invisible displacements). Elaboration of "representative groups."

STAGES 1 AND 2: PRACTICAL AND HETEROGENEOUS GROUPS

Until the age of three to six months, that is, until the prehension of visual objectives, the child's activities merely lead him to analyze the content of sensory images: forms as a whole, figures, positions, and displacements. Instead of objects, there are perceptual clusters corresponding to gustatory or "buccal" space of Stern, visual space, auditory space, tactile, postural, kinesthetic spaces. Although there are some interconnections by coordination of sensorimotor schemata, these spaces do not constitute a single space in which objects may be located. These are spaces momentarily created by actions.

Henri Poincaré (The Value of Science), in his analysis of the concept of space, considers as elementary the distinction between changes of position and changes of state. The former are distinguishable because they can be corrected by body movements which lead a perception back to its initial state, while the latter cannot. Undoubtedly this differentiation is gradually made. But is this distinction primitive? Does motor adaptation to displacements imply an immediate perception of groups? By stating that the distinction between changes of position and changes of state is present at the very first, Poincaré has endowed primitive consciousness with postulates presupposing an already refined mental elaboration. There is nothing to prove that sensorimotor adaptation to displacements immediately brings with it the concept of changes of position, and there is nothing to prove that an activity leads the subject to perceive displacements as such. (1) In order to distinguish a change of position from a change of state, the subject must be able to conceive of the external universe as being composed of substantial and permanent objects; otherwise the act of finding a displaced image would be confused with the act of recreating it. (2) The external universe must be distinguished from personal activity. If the perceived phenomenon and the acts of accommodation necessary for its perception were not dissociated, there could be no consciousness of the displacement. Poincaré says we can tell changes of position by muscular sensations. But to thought which has not distinguished an external world from an internal world, impressions of all kinds may be attached to all movements, and the subject will be unable to know when it is things or it is himself to which the displacement should be attributed. (3) To conceive of displacements is tantamount to locating oneself in an external spatial field, independent of action. But these three conditions are not satisfied during the first stages, so that even if the subject's actions constitute groups from the point of view of the observer, the subject himself is unable to imagine them as such. For to do this, the subject would have to view the objects as moving in relation to himself, each other, and certain landmarks, and to view his own displacements the same way.

Are the above conditions prerequisites for groups, or results of their appearance? They are developed in the use of practical groups, but are requisite in the elaboration of the conscious groups. Let us examine the different groups.

The first schema is that of <u>buccal space</u>. The displacements of the mouth in relation to objects constitute the simplest practical groups it is possible to observe in the child. We may distinguish: (a) displacements of the mouth in search for the nipple, (b) the reciprocal adjustment of thumb and mouth, and (c) the adjustment of objects seized for the purpose of sucking. Obs. 67, p. 106 sums up these factors, which are familiar from <u>O.I.C</u>. These movements are arranged in groups from the observer's point of view; for example, in trying to reach the nipple with his lips, the child corrects his displacements to the right by displacements to the left, and the mouth can approach the hand as well as the hand approach the mouth. But there are no perceived groups for the reasons stated above. Buccal space is a practical space which permits the child to rediscover positions, and adapt himself to forms and dimensions, but which does not allow him to apply such schemata beyond the immediate action.

The main practical groups resulting from <u>visual accomodations</u> come from (a) following movements of translation, (b)finding the position of objects, and (c) estimating distances in depth. For (a), see <u>Origins of Intelligence in Children</u> (<u>O.I.C.</u>) Obs. 28-32 showing how the child watches movements of translation or fixes his glance on a stationary object by correlating the movements of his eyes or head. For (b), Obs. 2, 5 in previous chapter show how the child can find an image again after losing it because the object moved too rapidly or after looking elsewhere. (c) presupposes, in addition to other factors, a relating of displacements and parallax. But the child is not aware of groups: in Stage II he can only rediscover the object in the extension of accommodation or return to the object in the initial position in which he perceived it (Obs. 5). From the subject's point of view, he finds his own initial position, not an object. There is not yet real localization of objects in depth, for only during Stage IV (9-10 months) does the child search for one thing behind another, and it is only prehension of objects which in Stage III leads to acquisition of concepts of "in front of" and "behind."

Binocular convergence is not systematic until about nine months (although it begins to appear at the end of the first month.) There may be tactile depth, but this is purely practical, and does not correspond to any visual perception.

In conclusion, the first two stages are characterized by the purely practical nature of the groups and the relative heterogeneity of the different spaces.

STAGE 3: THE COORDINATION OF PRACTICAL GROUPS AND THE FORMATION OF SUBJECTIVE GROUPS p. 113

This stage begins with the coordination of sight and prehension, and ends with the search for hidden objects. The new element is the coordination of various practical groups, and the essential feature in such coordination is the development of prehension. Without as yet detaching groups of displacements from the actions themselves, prehension nevertheless makes it possible to transcend the level of the practical group and to form the subjective group.

Prehension brings about two essential acquisition: (1) The child begins to make use of the relationships of things among themselves in contrast to the simple relations of things with the functioning of organs. (Secondary circular reaction of O.I.C.). (2) In the process of intervening in displacements and spatial connections, he begins to watch himself act and can begin to relate movements of his own to those of the environment. The projection of the practical group into the perceptual field determines what we shall call the subjective group, but does not yet produce objective groups, for, beyond the immediate action, the child does not take into account the displacements of objects.

First we describe the elementary groups, half practical, half subjective, which merely extend the purely practical groups of Stage II. The simplest of these correspond to what was called interrupted prehension with regard to the object concept, in Obs. 13-15: having dropped an object, the child searches for it in the extension of his earlier movements of prehension. The groups implicit in the adjustments of the hand are not perceived, for if the movement of prehension is not sufficiently delineated before the object disappears, the child behaves as though the object did not exist. Obs. 69-70 show transition between purely practical groups (the child experiences only impressions of "no longer holding" and "holding again") and subjective groups (he objectifies his action enough to perceive it partially from without but not enough to make a real "object," so that the object is a sort of extension of his action). As in Obs. 70 the child watches what he is doing, but perceived space does not yet consist in a system of relationships between objects, but is only an aggregate of relations centered on the subject. If an object disappears it is only followed by the glance if it has moved rapidly enough to produce strong movements of accommodation, which are extended.

At this stage we may examine the formation of groups related to buccal space which are formed in coordination with tactile and visual space and which determine the movements of rotation. When objects brought to the mouth have a side particularly favorable to sucking, the child is able to turn them around to find the "good part." We must investigate the psychological level of this group. In Obs. 76-77, ages 4-6 months, we see first how the child moves the object by chance, trying to suck it, and retains only those of his gropings which are successful. Later he coordinates the movements of his mouth and hands, but it is still empirical groping made more systematic through this coordination. Next we see the role of sight in the reversing group. In Obs. 77 the child grasps the object in such a way that he chances to see the good part and immediately tries to steer it to his mouth. In so doing he happens to catch it on his arm, and he sees this happen. He pulls harder and harder, but does not correct the movement by turning the object in the other direction. The only systematic reversals of which the child is capable are observed half-reversals (bringing to himself a side of the object which has already been perceived), any total reversal (bringing to himself intentionally the reverse side of the object) being as yet impossible. If the same rattle is repeatedly given to the child with the handle (which can be sucked) facing away, the child never turns it over right away, but only when he perceives the handle.

Here are groups from the point of view of action, but the child can not imagine the reversal. In relation to the mouth there is not true rotation; there is simply a special position (contact of the lips with the nipple of the bottle or with the handle of the rattle) and the child finds it again without imagining how he does so, by simple motor accommodation to the object. So long as sight does not direct the movements there is no perception of rotation from the tactile point of view either. When a child of this age holds an object, he passes it almost constantly from one hand to the other, and in the course of this manipulation he notices whether the side he brings to his mouth is pleasant to suck. The hand does not intentionally turn the object over; it adapts it to the mouth, and it is to the observer that this adaptation consists in a rotation. For instance, in turning a stick to suck it (Obs. 76) the child is elaborating a schema analogous, though slightly more complex, to that of thumb-sucking and the primary schemata without perception of the movements of the object. (He may try to suck the middle part and then shift the stick, or he may follow the edge of the stick with his lips, and make mutual movements of head and hands, guided by impressions of mouth and fingers.)

When the child looks at the object which he turns over, he perceives different sequential aspects of it. In later stages this will lead him to rotate the object systematically to study its outlines, and from this point on we may speak of an objectification of the group of rotations. But at present the child either searches with eyes or mouth for the special side he sees, or else he studies impartially all that the object has to offer by motor manipulations, and in neither case perceives the displacements themselves, arranged in groups. Obs. 78-79 show that systematic reversal occurs in this period (ca. 7 months) only after the nipple has been wholly or partly perceived, and that it is not the motor or technical difficulties which stop the child when he no longer sees the right end. For, if he sees the nipple, he can perform complicated motor adjustments to steer the nipple to his mouth; conversely, he can gaze at the bottle held vertically in front of him, his glance oscillating between the nipple and bottom, continuing the same kind of study after E has turned the bottle over, yet when the bottle is presented to him immediately thereafter with the wrong end facing him, he tries to suck and becomes discouraged without trying to turn the bottle around. He has understood nothing, despite extensive examination of the object when it was completely visible. In another example, the bottle is held at some distance and then brought towards the child slowly; so long as he sees the nipple he holds out his hands, but as soon as the bottle is turned so that the nipple disappears he begins to howl and withdraws his hands. Obs. 79 shows that when the child turns an object over, and transfers it from hand to hand, etc., he is doing it for the sake of the movement, not to see the invisible side; for instance he may in transferring it from hand to hand separate his hands to lengthen the trajectory, showing motor level and not exploration of the object as such.

Summarizing, we see that so long as he sees the desirable parts he can rotate the object. This group is both practical and subjective, since it is accompanied by perception of the movements of the object and perhaps of the hand. But it is not objective, since he cannot conceive of a complete rotation leading to a search for the reverse side. Instead he may just strike the object or suck it as though the right side had been reabsorbed in the object, and had ceased to exist spatially so as to remain simply at the disposal of the appropriate actions. This is consistent with what was seen in O.I.C. concerning the absence of true explorations during the third stage. (He is only interested in exercising the schemata he already has in relation to the object.) It is only in Stage IV that he begins to explore the object in order to learn its nature, and only in Stage V that he begins to experiment upon it (tertiary circular reactions). At the present stage the rotations are only secondary reactions. (A secondary circular reaction utilizes relationships of things to each other, and not merely to the functioning of organs.) We see this in O.I.C. Obs. 98-109, and the present work Obs. 80, in which the children shake objects held in the hands or shake the bassinet hood by shaking dolls hanging from it, or use a string to shake the hood. Contrary to the case in which the eyes, hand, or mouth alone follow an object without knowing their own spatial displacements, the behavior patterns in the present observations presuppose a perception of the group. The rattle is treated as an object endowed with backwards and forwards movements, which the child corrects and directs. These interventions are known to the child through the sight of his hands, and in each case the child perceives movements that can be repeated with the manipulated objects and perceives the movements as governed by his action. Partial spaces, however, have not yet been extended to include the whole subject, and the object has permanence only relative to the action itself. The groups which define the present level do not yet concern the interrelations of objects; they merely connect a subject partly unaware of himself with objects that are semi-permanent and not spatially arranged in relation to each other. We may study this by analyzing the spatial relations of objects. For true objectivity, objects would (1) be arranged in depth, and (2) acquire constant size and form. (p. 134)

(1) Depth. It is relatively easy to determine the extent to which the child accommodates his eyes and hands to depth and how he behaves with respect to objects arranged according to the third dimension. The problem is then to discover how, from these actions bearing on distances, he will draw an awareness of the third dimension capable of interconnecting things in a spatially organized universe. One must distinguish the tests in which the subject could not succeed without our spatial representation (tests related to hidden objects, for instance) from the current behavior patterns common to all levels of spatial perception (looking at or grasping objects at different distances, etc.).

The important innovation in the development of practical groups is the coordination of prehension and sight. In preceding stages the eye already accommodated to distances, but only at the present stage is this used to regulate the hand. We may now analyze accommodations of the hand and total displacements of the child. (Obs. 81-84) On the one hand, it seems that the child distinguishes in general what he can and cannot grasp; in this respect he evidences measurement of depth. Moreover, he learns to approach the distant object (Obs. 81, III and Obs. 83, III), which reveals the same thing. But on the other hand, he also behaves as though he did not know how to measure the presenting distances with certainty; sometimes he tries to obtain objects that are out of his reach (Obs. 82, II and 84, II), he commits continued errors with regard to near objects (Obs. 82, III and 84, III), and he often believes he is moving closer when he remains in place (Obs. 82, IV). To what extent does such behavior prove that the child considers the difference between near objects and distant ones as a difference in depth? We may see that perceptual knowledge corresponding to practical accommodations to depth must be interpreted as a function of subjective groups and not of objective depth.

First we note that the impetus to grasp objects is not solely a function of their distance. There are numerous objects which the child does not try to grasp, even when within his reach. There are familiar things which he has always looked at in a practical situation which has excluded prehension (e.g., the bottle in Obs. 82, I and 84, I); and there are things that are unfamiliar or that are presented in abnormal circumstances (Obs. 83, I). We may postulate that distant objects, while being recognized by the child, seem to him to be precisely what they are in crude and uncorrected perception--diminished, distorted, and linked to a context in which direct prehension has never intervened. It is very possible that, without awareness of distance as such, the child does not try to grasp them merely because they are different from when he touches or grasps them ordinarily. For we know that even though the child has begun to coordinate prehension with sight, he begins timidly, and generalizes the pattern only gradually. Besides, it is precisely around 6-7 months, rather than 3-6 months that the child begins to wish to grasp distant objects, as though regression had occurred, whereas there is merely generalization of prehension. Thus it is impossible to draw from the difference in reactions toward near and far objects a decisive argument in favor of correct knowledge of distances. The contrast between near space and distant space is a practical distinction, near space being that of objects of normal size and shape on which prehension has already been brought to bear, and distant space that of diminished or distorted objects, situated in a context in which secondary reactions and "procedures for making an interesting spectacle last" have shown themselves to be immediately frutiful (i.e., "magical" procedures rather than prehension).

Then we may suppose that the child's whole space considered from the point of view of distances is analogous to the celestial space of immediate perception: a fluid mass without depth (although the eye accommodates to various distances), traversed by images which interpenetrate or become detached without laws. A number of practical groups intervene in relation to the movements the child makes in the first two stages to follow or rediscover interesting images. With the coordination of prehension and sight in Stage III, the movements of the hand give the child the opportunity to make experiments with depth, and subjective groups are added. Thereafter, space becomes dissociated into two zones, "near space" accessible to the construction of subjective groups related to depth, and "distant space" which inherits all the remnants of the space of the first stages. Before the prehension of visual objects, it may be as though all of space were like the sky. When he begins to grasp, this sphere expands little by little, and distant space seems like a neutral zone in which prehension is not yet ventured. Only after the establishment of planes of depth makes it possible to adjust objects in near space in relation to prehension could distant space really seem distant, that is, a background in which relative distances remain undiscernible.

Piaget then refers to examples showing the subjective nature of the concepts "in front" and "in back," (Obs. 81, III, 83, III, 82, IV, Obs. 26-27 in relation to screens, and O.I.C. Obs. 103). The child learns how to grasp, hence to localize, objects in relation to himself, but he has no definite concept of the position of objects in relation to one another; in the next stage he will still look for objects in two places at once, or will look for them in their old place without taking into account their sequential positions. For objective knowledge of displacements, the child must take into account the number of objects separating the subject from the perceived object, the superposition of these objects, and their relative movements when the subject is displaced.

(2) Permanent form and dimensions. This requires and is required by the constitution of an objective group of displacements. H. Frank showed that the

concept of constant size is acquired during the next stage, as early as 11 months. This occurs at the same time as the beginning of searching for things behind screens. But during Stage III there seems to be no assumption of such permanence. When he turns things he holds in his hands (Obs. 76-77, 21-25) he does not yet seem to be exploring their shape for its own sake. When he grasps a part in order to have a whole (Chap. I, Obs. 21-25) he is completing a whole but not attributing constant spatial qualities to that whole, as we see from the absence of any search connected with the reverse side of objects (Obs. 78-79).

In conclusion, during Stage III the subjective group is superimposed on the practical group wherever the child perceives that his action can introduce or rediscover a repetition in the images perceived. It extends the practical group only by adding perception of the group, but the group does not yet concern the interrelations of objects. For the establishment of objective groups there is required the existence of substantial objects, the differentiation of external displacements from his own movements and the externalization of spatial relations so that the subject is capable of locating himself "in" space. But he does not know permanent objects, he probably cannot tell movements of his head and eyes from those of the environment (Obs. 88-91 below, in which the child investigates the effects of head movements, looking upside down, etc.), and may have localized hand movements in space, but certainly not positions and movements of the whole body. Space is imbued with an egocentrism unaware of itself.

STAGE 4: THE TRANSITION FROM SUBJECTIVE TO OBJECTIVE GROUPS AND THE DISCOVERY OF REVERSIBLE OPERATIONS p. 152

The starting point for this stage is the application of familar means to new situations. This behavior consists not in constructing schemata differently, but in applying and intercombining them in a new way. Up to now the primary or secondary schemata formed new global totalities, each of which was applied as a unit in the presence of suitable objects and was generalized to the extent that new objects could be directly merged with the old ones. Now he applies them to new situations or subordinates them to a complex act. Two consequences are that accommodation to things becomes precise, and that the schemata adapt to each other and cease to function separately. This means that relationships are being woven among the things themselves, instead of being wrapped up in the totality established by the action. When the child removes obstacles between himself and his objective, or uses someone else's hand in order to act upon things, he intercoordinates not only his schemata, but the objects themselves. The main characteristics of space of this stage are the discoveries (a) of reversible operations, (b) of the constant size of solids, (c) of the perspective of relations of depth, and (d) of the permanence of the object masked by a screen.

The child begins to search for moving objects behind a screen even when they have disappeared without extending a prehensile movement already begun. This gives rise to the spontaneous behavior of hiding and uncovering toys, feet, etc. (10-12 months). This is the first instance of the formation of an objective group of displacements, although an almost trivially simple one. But he still does not take sequential displacements into account. Space is externalized to the extent that the action must be conceived by the subject as being inserted into a ready-made world and no longer as continually engendering that world, but permanence still remains linked to the action itself. Constancy of shapes and dimensions also is acquired at this time. Obs. 86-87 show the child studying the changes in appearance caused by moving the object systematically. The child studies the fact that an object whose tactile dimensions are constant varies in visual shape and size as it is moved. The child also moves his head towards and away from the object, and may discover the possibility of changing appearances by moving his head, i.e., the relations of perspective in rudimentary form. Obs. 88-91 trace these movements from the second stage, when they are used as procedures for acting on the object itself (which is not distinguished from the view or the action) to the fourth stage, in which the groups are not merely practical or subjective, but tend to become objective, and in which the movements may be viewed as attempts to understand. At the same time it appears that the child begins to be able to view his movements as displacements distinct from the displacements of objects. Support for these assertions is given on pp. 160-162. But these groups are almost trivial in that they consist only of one operation and its inverse. Similar progress is noted with regard to rotations. Whereas, during the third stage the child turns an object over to rediscover a special side which he has perceived during the rotation, or just in order to turn it over, during the fourth stage, he learns to turn things over in themselves, and thus acquires the concept of the "reverse" side of the object and consequently of its constant shape. Obs. 92-93a show examples of intentional rotation with exploration of the object and search for the reverse side. For instance, if the bottle is presented with the nipple facing away and invisible, the child immediately displaces the wrong end of the bottle, while looking beforehand in the direction of the nipple. But again, this is a mere system of reversible single operations. It is only when the child holds the object in his hands that he is capable of reversing it. As we shall see later, the child at this stage does not yet know how to turn objects over in relation to each other. Obs. 94-95 show progress with regard to perception of movements of translation. One observes intentional displacements of objects designed to study their movements. We recall that during Stage III the child is capable of following rapid movements of translation, but on condition that they merely extend movements of accommodation; once it has left the trajectory thus fixed in advance, the object is no longer sought or followed by the glance (Obs. 71-72, pp. 118-119), whence the difficulty of marking the position of the vanished object (Obs. 74-75, p. 122). Henceforth, the child whose eyes follow a rapid movement and who loses sight of the object searches for it whatever its trajectory may be and independently of the direction of the initial movement of accommodation. These observations leave no doubt concerning the fact that the child distinguishes changes of position from changes of state, as he was previously unable to do. He is aware of certain movements of his head and trunk, and no longer only of those of his hands, and he attributes more permanence to objects. He cannot yet arrange sequential displacements in time, but he does apprehend directly perceived displacements in the form of objective groups, perceiving permanent objects of constant shape, which follow autonomous trajectories. But again, these are almost trivial groups restricted to reciprocal movements. The above behavior is closely related to the child's reactions to interrupted prehension, illustrated by Obs. 96, p. 169. During Stage III the child can follow the lost object with his hand, if the trajectory of the object follows the movement of prehension already outlined, or a simple movement of the forearm (Obs. 69-70, p. 115); the child in the fourth stage, however, really searches with his hand for the vanished object.

Depth perception in Stage III involved regarding space as comprising two regions: one beyond the field of prehension and the other defining the field itself. In the fourth stage the child begins to displace himself and to approach objects, and moreover, he searches for objects behind other objects and thus inaugurates an effective ordination of the planes of depth. Little by little this progress pertaining to near space is extended to distant space. (a) Generalization of attempts at prehension and progress in motility eliminate barriers between the two spaces; the child can grasp increasingly distant objects, and these objects acquire more shape constancy and permanent existence. (b) Ordination in depth and search for masked objects are generalized to apply to more and more distant space (Obs. 97-99, pp. 171-173). (c) The systematic investigation of perspective by moving the head allows the child to use parallax as a cue for arranging objects in depth.

However, the acquisition of reciprocal operations does not entail the formation of complex objective groups, or, consequently, of a motionless space in which the subject would place himself in toto. The subject continues to locate things in relation to himself and not in relation to each other; the object has, in a way, an absolute position, that in which the child attained it the first time. Space is not a homogeneous environment for the relative displacements of objects, but still consists in qualitative aggregates arranged as a function of the action, and as images as a whole. A consequence is that the child cannot perceive his own displacements outside the simple groups of reversible operations. He may be able to move his head or hands, but not his whole body in a systematic way, and he may not understand the relation between his own movements and those of objects, even when reversible operations would suffice. E.g., (Obs. 100, p. 175) a child's head is caught between a wall and a taut vertical string, but she does not know how to extricate herself by backing out. A second consequence is that the child can only make very elementary relationships among objects from a spatial point of view. For instance, when the child pushes back an obstacle in order to reach the object, or brings someone else's hand near an object on which he wishes it to act, or even (O.I.C., Obs. 130) drops an object over a basin so that it may make a noise on striking it, etc., he confines himself to utilizing simple relations such as "moving away from" or "moving near to." Thus the groups remain midway between subjective and objective. More complex relations await tertiary circular reactions with active experimentation during the fifth stage.

A characteristic example of this situation is that of the relations of objects placed upon one another (pp. 176-182). Although the child can search for an object under a screen and move the object toward or away from another, in reality all the behavior patterns of this type remain relative to the action in progress. The relation between the objects is not given for itself between two independent objects. Two groups of facts support this. (a) Szuman and Baley reported the difficulty experienced by the child in grasping an object that is "upon" another when he perceives both objects at once. S. Szuman ("Observations on syncretic perception in children," Archiw. Psychol. 2 (1927) No. 1) has shown that 7-month old children do not know how to grasp a small object placed upon a support; when they try to put this object in the mouth they seize the support and try to swallow the object along with it. Baley ("Behavior of children and animals when confronted by objects placed upon a support," Polsk. Archiw. Psychol., 1 (1932) No. 4) found that some children give up grasping the desired object as soon as it is placed on a support. Both reactions are found in primates. A small support is apt to invoke the positive reaction, and a larger support the negative reaction. In the case of the negative reaction the animal often presents curious behavior, "as though he were afraid." Piaget found the same facts (Obs. 101-102) during the third and fourth stages, and concluded that the general difficulty of conceiving relations independent of the child's actions keeps him from realizing that two objects can be independent of each other when one is placed upon the second. The negative reaction may consist in striking the support and giving up the attempt at grasping, or the child may grasp the support and not the object. If the support is extremely large in relation to the object it is regarded as a neutral base, and the child succeeds in grasping the object. The child also has difficulty trying to balance one object on another, e.g., trying to place a thimble on its open end upon a box so that it will not roll. Everything takes place as though she did not know how to turn one object over systematically upon another, whereas she well knows how to do this when it is in relation to herself alone. (b) The second supporting facts show that the child cannot bring an object toward himself by utilizing as an intermediary the support on which the object is placed (Obs. 103). Curiously, the child remains incapable of such behavior (except in the event of special training), whereas in the case of the supports of very limited dimensions he does not dissociate the object from the one on which it is placed. When the support is small it is perceived or conceived as being one with the object; when the support has a large surface, on the contrary, it constitutes a sort of neutral base, and the child does not understand that the object is upon it and that movement of the support will entail movement of the object.

STAGE 5: "OBJECTIVE" GROUPS p. 183

The essential advance of the fifth stage in the construction of the spatial field is the acquisition of the concept of the relative displacement of objects, in other words, the elaboration of objective groups of displacements in the midst of a homogeneous environment. In regard to the object concept, the criterion of the appearance of the fifth stage is the child's success in noting the sequential displacements of the thing he seeks, an ability prerequisite to arranging the movements of objects in a collective system that insures the homogeneity of the spatial field.

To study this progress, we first note the behavior which consists in throwing an object outside the visual field and in <u>finding it again by a path different from the one that was followed in hiding it</u>. It is no longer a matter of a simple reversibility of movements, but complementary movements linking one another. Here we see the elaboration of objective groups. (Obs. 104-5, at about one year of age.) The sequence is: reversible operations (hiding and finding by the same route); then finding by a different route, this group being related to the body itself; and finally, the objective group, entailing relations established among things as such (seeing an object disappear behind the child's mother and searching for it on the other side). This discovery and utilization of relations among objects themselves leads to the development of geometric relations.

The most typical and important behavior pattern in this regard is the child's experimental study of <u>visible displacements</u>: carrying objects from one place to another, moving them away and bringing them near, letting them drop or throwing them down to pick them up and begin again, making objects slide down a slope, and in short conducting every possible experiment with distant space as well as near space (Obs. 106-108). Obs. 109-110 concern experiments with <u>position and equilibrium of bodies</u>. Once more we begin to see object-object relations and not just object-subject relations. Obs. 111-113 reveal that at about 15 months of age the child puts solid objects into hollow objects and to empty the latter to recover the former, in a series of experiments to discover the <u>relation of contents to container</u>. At first the groups are the simple reversible ones of putting something in and taking it out. In this form they barely transcend Stage four, since, as we have seen in connection with the object concept (Obs. 60-63) it is enough to increase the number of displacements for the child to return to the concept of a special position. But then the group becomes complicated: emptying the container in order to pick up the contents on the floor, etc. Moreover, e.g., when one child empties at one stroke the contents of a basket which she filled bit by bit she sums up in a single operation a possible series of detailed operations. Obs. 114-115 show progress in <u>reversals</u> or <u>rotations</u>. Whereas previously the child confined himself to turning objects only in relation to himself, now he learns to turn them over in relation to other objects. Finally, the child begins to gain awareness of <u>his own movements</u> as displacements of a whole. This does not yet mean that he places himself in relation to other bodies in a system of reciprocal relationships, but that he purposely displaces himself in the direction of desired objects and thus acquires the ability to elaborate groups more complex than before, especially with regard to depth. Obs. 116-117 show how the child can reach the same positions by walking along different paths, and can walk methodically to a desired position. But the objective groups discovered during this stage remain limited to the displacements directly perceived and do not yet include any displacement simply imagined. The child cannot take account of displacements produced outside the perceptual field or to locate himself in relation to objects. When the child is trying to pull an object towards himself by means of a support, he may pull the "support" even if the object is only alongside it (O.I.C.Obs.150). If he pulls an object with a string, he may not pull the string taut (O.I.C.Obs.154). If he tries to pull an object through the bars of his playpen he does not know he must turn the object to present the narrow side (O.I.C.Obs.162-166). He cannot foresee relationships, but confines himself to organizing them after the event. Obs.118-120 give examples of these difficulties of representation. In one example the child puts toys down the neck of her clothing. They fall down to the legs, but she does not know she can remove them there. When Piaget removes them, she is astonished at the return of her toys, and immediately looks down the neck of her garment as if to verify that they are no longer there. The child may be conscious of his own movements and able to guide himself to distant objects, but he cannot solve a problem which requires representation of his position as an object among objects. Obs.121 and O.I.C.Obs.168-169 show the child tugging on objects he is standing on, finally giving up the attempt to pick up the objects. In Obs.122 the child raises one foot out of a ditch and tries to pick up the other foot with her hands.

STAGE 6: "REPRESENTATIVE" GROUPS

During the sixth stage the child becomes capable of rediscovering a hidden object after several sequential displacements, even if some of these are effected outside the visual field. Hence there is representation of movements, whatever the method of production of these representations may be. This presupposes the representation of space, for otherwise the perceptual universe remains incomprehensible. Spatial representation is required to arrange the various displacements into a coherent whole (which entails filling in unseen movements and allowing for apparent movements). In addition the child needs representation of himself as an object in spaces. When the child invents new means through mental combinations, as analyzed in O.I.C., he mentally combines the spatial relationships of objects, as, for instance, when he rolls a watch chain into a ball before introducing it into a narrow opening (O.I.C.Obs.179) or tilts up a stick before pushing it between bars (O.I.C.Obs.178). This capacity for spatial representation has the immediate effect of inducing the child to invent detours, that is, itineraries that allow for obstacles. This pattern of the detour seems the most typical behavior pattern acquired during the sixth stage. It presupposes representation of spatial interrelations and the elaboration of representative groups. Examples of spatial detour behavior and representation of spatial relations are given in Obs.123-127. These illustrations show behavior presupposing representation that anticipates the steps to follow and an itinerary which either is not visible in its entirety or else entails a play of complex relations. During the fifth stage, on the contrary the child limits himself to following the path that is directly perceived (Obs.116-117) or if he goes around an obstacle to search for something, he confines himself to adopting the path already followed by the object which has just disappeared. Obs.126-127 show that the child can point in the directions of the customary places of various people, and can point out the direction of home on the outgoing and return legs of a trip. These observations show how the child, having become capable of representation, tends to arrange different spatial aggregates in relationship to each other. The above descriptions of pointing show that the child places himself in a universe which has become stationary and which includes himself. The initial egocentric space is turned around, and the subject is aware of his displacements among permanent objects which have movements independent of the subject.

THE MAIN PROCESSES OF THE CONSTRUCTION OF SPACE

The problem of empiricism and nativism is badly stated; the reality of space is in its construction and not in the extended or unextended character of sensations. In studying these constructions one must accept these facts: the very functioning of biological and psychological assimilation entails a priori an organization by groups; and from the beginning of their activity the organs of perception apply this organization to the displacements they perceive. Groups and relations are inherent in any operations which the infant may perform. They may be said to be involved in reflex turning of the head toward light as early as the sixth day of life. But how does the child, starting from a space completely centered on his own activity, manage to locate himself in an ordered environment which includes himself as an element. We see two interconnected processes: (a) progressive structuring of the spatial field, and (b) desubjectification of its elements.

With regard to structuring, we observe in Stage 1 the behavior patterns of sucking, sight, etc., reveal a hereditary coordination of movements in space, but without spatial intercoordination. In Stage 2 the primary circular reactions permit the child in each of the buccal, visual, tactile, kinesthetic spheres to follow or rediscover the habitual perceptual images by means of movements grouped in coherent systems superposed on the reflex systems. The perception of space is therefore still reduced to that of certain movements of bodies in the respective fields of various sensory organs, and the child imagines neither displacements external to those fields nor movements of the body itself, not even coordinating in a single environment the various spaces thus sketched. With the advent in Stage 3 of secondary circular reaction, that is, the coordination of sight and prehension, the structuring of space achieves the coordination in a single system of the different practical spaces, and the formation of groups in the field of perception. But coordination does not go beyond perception, and the field does not include the body itself as such, but only manual activity. In Stage 4, with the intercoordination of secondary schemata, space goes beyond immediate perception, since the child can search for vanished objects. But by failing sufficiently to detach the object from personal activity this structuring extends only to the reversible groups and does not yet concern the free motion of bodies or the body itself conceived as an object. In Stages 5 and 6 the structuring extends to the aggregate of displacements which have been perceived sequentially and then to those which the intelligence is able to reconstitute deductively without having seen them. Reciprocal relations are thus established among bodies in motion, and the body itself is conceived on the same plane as other objects.

To the extent that activity is regulated by global schemata, spatial coordination operates only between the subject's movements and objects which are in their immediate extension. To the extent, on the contrary, that the schemata become sufficiently mobile to combine among themselves in many ways, spatial relations are established among objects on the one hand, and affect the body in toto on the other. This means that the true nature of space does not reside in the more or less extended character of sensations as such but in the intelligence which interconnects these sensations. The sensations themselves exist only as functions of perception of the totality linked with mental assimilation, and thus all is the result of intellectual activity, and space cannot be conceived as a reality separate from the whole of the work of the mind.

The structuring of space just outlined can be described from the point of view of behavior. Spatial desubjectification and consolidation are, on the contrary, essentially related to the acquisition of consciousness, which we must try to construct through cur knowledge of the baby's behavior. The sensory images are originally uncoordinated with each other from the point of view of space. Only the movements which accompany such perceptions are, each in its own realm, hereditarily organized in mechanisms constituting so many practical spaces. Thus consciousness can at most be the feeling of being able to rediscover certain perceptual images which do not have a stable relationship either among themselves or with the subject. At first there is neither external nor internal world, but a universe of "presentations" whose images are endowed with emotional, cenesthetic, and sensorimotor qualities as well as physical ones. This primitive universe constitutes thenceforth the child's self as well as the objective of his actions. Hence there are as yet neither substances nor individualized objects, nor even displacements, since without objects changes of position cannot be distinguished from changes of state; there are only global events connected with movements of the body, hence with kinesthetic and postural impressions. But gradually as space is developed the situation is exactly reversed. Space encompasses all the images in a single environment. These images are detached from the activity itself, and are externalized and interrelated, and are consolidated into permanent and substantial objects. Above all, the child discovers his own body and locates it in space with other objects, establishing a totality of reciprocal relations between his own movements and those outside. This development goes hand in hand with the formation of substantial objects, for without these, the extension inherent in the various perceptions could not be that of a space external to the self.

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CHAPTERS III AND IV: THE DEVELOPMENT OF CAUSALITY AND THE TEMPORAL FIELD

These notes will not cover Chapters III and IV, although they are inseparably connected with the earlier chapters, just as we have seen that the development of the object concept and the development of space are necessary to each other. In order to conceive of objects and space, the child must separate them from his own activity, and must become aware of the effects of his activity. Thus object and space require awareness of causal relations. Piaget considers primitive causality to be a feeling of efficacy linked with acts as such, with the reservation that such feelings are not considered as coming from himself, but are localized in the perceptual aggregates constituting his actions. Thus, there are a collection of centers in which the child localizes feelings of effort or efficacy. With the advent of secondary circular reactions the child uses his actions in a magic-phenomenal way to produce desired results. With the development of independent objects and their relations he begins to be aware of true causal relations.

The gradual development of the ability to group actions into temporal series and to form a mental representation of time is linked in like fashion with the groupings of the sequences of actions required in order to develop the notions of stable objects and their relations.

Some partial notes on

7. THE CHILD's CONCEPTION OF SPACE, by Jean Piaget and Bärbel Inhelder. Transl. by F.J. Langdon and J.L. Lunzer, N.Y.: Humanities Press, 1956.

PART ONE: "TOPOLOGICAL" SPACE

SUMMARY

Chapter I studies how the child recognizes objects by touch alone ("haptic perception," Fr. "stereognostique"). Chap. II studies drawing of geometrical figures. Both chapters show that the child starts by building up and using relationships such as proximity, separation, order, and enclosure. The remainder of Part One is devoted to a more detailed study of these relationships. Chap. III deals with order, entirely from the point of view of spatial proximity and separation. Chap. IV treats enclosure or surrounding, in terms of the understanding of them derived, for example, from a working knowledge of knots. Chap. V treats continuity and the gradual subdivision of lines and surfaces into smaller units, and ultimately into points.

CHAP. I: PERCEPTUAL SPACE, REPRESENTATIONAL SPACE, AND HAPTIC SHAPE PERCEPTION

Evolution of spatial relations proceeds at two levels: perceptual level and level of imagination or thought. The idea of space develops under the influence of motor and perceptual mechanisms, but it is incorrect to assume that representational and geometrical ideas are merely a copy of existing sensorimotor constructs. Even after sensorimotor space has provided experience of straight lines, angles, circles, squares, systems of perspective, etc., representational thought begins by ignoring metric and perspective relationships, but relies only on proximity, separation, order, enclosure, etc., applying them to the metric and projective figures yielded by perception at a higher level.

Later, representational activity is reflected back onto perception. Thus after representation can arrange spatial figures in a coordinate system, perception begins to do the same.

This chapter will review sensorimotor and perceptual space. Then the study of haptic perception -- the tactile recognition of solids -- will give the opportunity of observing the development from perception to representation, and the translation of tactile perceptions and movements into visual images in children between the ages of 2-7 years. Drawing will show the transition from visual perception to ideomotor representation (motor activity as the direct expression of an idea). These studies provide a natural introduction to the study of representational space.

Section 1: Perceptual or sensorimotor space.

1. Spatial perception prior to representation.

This is described in Chap. I-II of Construction of Reality in the Child (C.R.C.). Children don't have all the mature spatial perceptions from the start. At 7 or 8 months a child has no idea of the permanence of objects, and does not reverse a feeding bottle presented wrong way around. Size constancy hasn't reached its adult stage even at 8 years. So perception of space involves a gradual construction.

We may sketch this construction during three main periods of sensorimotor development, from birth to beginning of representation. 1st period: I. pure reflexes, II. primary habits. 2nd period: III. "secondary circular reactions" (beginning of manipulation of objects, about 4-5 months), IV. first fully intelligent behavior (to about end of first year). 3rd period: V. "tertiary circular reactions" (beginning of experimentation), VI. "internalized coordinations" (rapid comprehension of novel situations).

First period. Absence of coordination between various sensory spaces -- in particular, between vision and grasping. Naturally, neither permanence of objects, nor perceptual constancy of size. What spatial relations are involved in this primitive perception (e.g., in the exercise of the reflexes of sucking, touching, seeing patches of light, and the earliest habits superimposed on these reflexes)?

1. Proximity. "Nearby-ness" of elements belonging to the same perceptual field. In accord with Gestalt school, except that this relationship alters as child grows older. For youngest child, this factor overshadows all others (like resemblance, symmetry). Later, he uses other factors.

2. Separation. Two neighboring elements may be partly blended, but now the child can dissociate them, and this is involved in the analysis of elements making up a syncretic whole (e.g., an object leaning against a wall). Separation and proximity develop hand in hand -- he establishes more and more numerous separations, but also takes account of different degrees of proximity operating over larger areas.

3. Order, or spatial succession, when two neighboring though separate elements are experienced. Examples: baby's gaze or touch passes over a series of elements, or sequences in which a door opens, a figure appears, and he is fed. Relation of symmetry.

4. Enclosure, or surrounding. E.g., B in ordered sequence ABC. Nose in face. This develops as the other relations develop -- at one time partial disappearance of object behind screen looks something like absorption by screen. At about 1 year, when child attempts to replace a ring which encircles a stick, he contents himself with pushing it against the stick (Origins of Intelligence in Children, p. 320, obs. 174).

5. Continuity. Develops with development of thresholds of sensitivity, relations of proximity and separation. For Poincaré continuity meant nontransitivity of perceptual equality.

During first few months perceived figures simply appear and disappear and exhibit a series of changing shapes in between, so that change of state cannot be distinguished from change of position. But after 5-6 weeks, when child smiles, baby can be seen to recognize a face.

Second period. Stages III, IV of sensorimotor development (4-5 to 10-12 mos.) are marked by coordination of vision with prehension, and the general coordination of actions. Perception is transformed by feedback from the systematization of movements under the guidance of vision. Thus this period is characterized by the building up of figures and the development of perceptual constancy of shape and size. In opposition to Gestalt theory, we believe that "good configurations" evolve. Stability of good configurations can just as well result from laws of sensorimotor activity as from those of pure perception. And cf. Brunswik's "empirical Gestalt": tachistoscopic stimulus object intermediate between symmetrical five-fingered object and ordinary hand is just as easily perceived as hand.

Before present period baby could not possibly perceive a straight line because (1) there is no objective line in the jerky and fluttering eye movements of the first few months; and (2) isolated lines do not exist in a baby's perceptual world, but must be abstracted from the boundaries of figures, yet these figures are not yet assumed to indicate permanent objects with fixed dimensions. But towards 8-10 months one sees series of explorations concerned with movements of objects and recognition of their shapes, tending to relate them to perspective (C.R.C.: pp. 167-9, 159-64). The line then takes on functional significance as a trajectory, as intersection of planes, as sole aspect of shape retained in perspective, etc. Along with construction of lines, circles, angles, etc., we begin to get constancies of shape and size. Thus projective and metrical relationships are interdependent. Shape constancy results from sensorimotor construction at time of coordination of perspectives. Previously alterations due to perspective looked just like actual transformations of the object. Not until 8-9 months does baby explore perspective effects of displacements, and it is just at this time that he can reverse a feeding bottle presented wrong way around. In this period, he begins to distinguish his movements from those of the object, and begins to find reversibility in movements.

Third period. From beginning of 2nd year there is systematic observation and inquiry, experimentation (Stage V), and internal coordination of relationships (Stage VI), and these activities react on sensorimotor space: Whereas 2nd period achievements relate mainly to shape and dimensions, 3rd period achievements consist in bringing out relationships of objects to each other. Thus group of movements is extended to movements of objects not directly perceived. (C.R.C.: pp. 187-195). Finally, in Stage VI we find the mental image, which makes possible delayed imitation, and as a result, the first attempts at drawing. (See Play, Dreams and Imitation in Childhood) (P.D.I.). Space, which was purely perceptual, has become partly representational, and this book will explore such representation.

2. Perception and movement: the role of "perceptual activity."

On sensorimotor level child can perceive and act in accord with metric relations, etc. Reproducing this advance at the level of representation takes several years, until 7-8 years of age. But both constructions have something in common: motor activity.

At each level, spatial thinking takes two forms: thinking of static patterns and expressing possible transformations. Does geometrical thought contemplate images of the first kind, or is it an active, constructive set of operations? It appears that at each stage the latter controls the former. Sensation and movement are not two independent psychological realities which may be associated: neither can occur without the other. A perception, like the sight of the feeding bottle turned wrong way around, is a system of relationships including potential relations as well as immediately present ones. Transformations continually increase in importance, as opposed to the original predominance of static forms. Perceiving a cube in any position, and in general, every perception, implies a sensorimotor schema which brings the total of previous constructions to bear upon the actual situation.

Centration: the part of the figure upon which vision is centered will be overestimated in size, according to investigations done jointly with M. Lambercier (Arch. de Psychol., 1943 et. seq.) The point of fixation is a matter of probabilities, and one can calculate the probable perceptual effect of a given shape relative to the different combinations of points of fixation. (Piaget, "Essai d'interprétation probabiliste du loi de Weber et de celle des centrations relatives," Arch. de Psychol. XXX, p. 95.) Müller-Lyer illusion comes from this. Visual perception itself is made up of a system of relations determined by probable movements of the eye, and we shall see the tactile analogue.

Section 2: The recognition of shapes ("haptic perception").

Representation completes perceptual knowledge by reference to objects not actually perceived. In addition representation introduces a distinction between signifiers and that which is signified. This chapter will primarily discuss the image, although this cannot be entirely dissociated from the concepts it serves to indicate. The image is most likely an internalized imitation)see Play, Dreams and Imitation in Childhood), and is consequently derived from motor activity, even though its final form is that of a figural pattern traced on the sensory data. As a result, the mental image benefits from the attainments of perceptual construction, eventually making use of ready-made forms like straight lines, squares, etc. But it must start with the simplest topological relations.

3. Recognition of shapes by haptic perception. Technique and general results.

The child holds objects behind a screen and must name them, draw them, or point them out from a collection of visible objects or drawings. He must thus transform tactile-kinaesthetic impressions into visual ones. The objects, depending on age and experiment are pencil, spoon, etc.; geometrical cardboard cut-outs of circle, ellipse, square, rectangle, rhombus, cross, star, swastika, semi-circle, semi-circle with notches along the diameter; irregular surfaces pierced by one or two holes, open or closed rings, two intertwined rings, etc. Also, figures of matchsticks or grooves. The first task is to translate into visual images. This is not new for the 2-4 year old. A 3-5 month baby (age of coodination of touch and vision) already makes visual perceptions of its hand or objects it manipulates correspond to tactile or kinaesthetic perceptions of the same. (O.I.C.) General development of imitation, especially of facial movements means the baby comes to know movements he cannot see (P.D.I.). The new problem is the second task, that of the construction of the image, and it is with this problem that the present study is exclusively concerned. As soon as objects become too complex to be recognized immediately by touch, the child is forced to make a tactile exploration, and thereby to construct a visual image of it. Drawing and identifying the object were found to be of about equal difficulty.

The results are summarized as follows. Stage I (3/6 to 4 years): familiar objects more or less easily recognized, but not geometrical figures. Stage II (4/6 to 6 or 7 years): geometrical figures are progressively differentiated. Stage III (6/6 or 7 years): synthesis of complex forms is achieved. Below 2/6 experimentation with hidden figures is not possible. Stage I may be subdivided: Substage IA: familiar objects recognized but not shapes -- even visual correspondence between shapes by simple superposition requiring practice. Substage IB (3/6 to 4): beginning of abstraction of shapes. But these shapes are not Euclidean but topological. Thus the circle and square cannot be distinguished because they are both closed forms, although they are distinguished from open forms. Neither straight lines nor angles are identified. Throughout Stage I tactile exploration remains relatively passive. The child simply grasps the object and responds to chance discoveries, such as poking his finger through the hole in the key handle. In Substage IB shapes are explored. IB-IIA: (4-4/6 years): crude differentiation of rectilinear from curvilinear shapes, while the two types are not differentiated among themselves. Representation by drawing is possible but lags somewhat behind identification by choice. Tactile exploration remains global but use is made of chance indications as they are encountered. IIA: (4/6-5 or 5/6). Progressive differentiation of shapes according to their angles and even their dimensions (circle-ellipse, square-rectangle). Drawing is beginning to catch up with recognition. Tactile-kinaesthetic exploration shows signs of a search for significant clues to identity. IIB: (5-5/6). Successive discoveries, attended by much hesitation, of rhombus and trapezoid. Crosses and stars begin to be differentiated but many errors still occur in the representation of complex forms. Exploration becomes more active, but it is not always systematic. III: (beginning at 6/6-7). Methodical exploration making use of reversible operations. Child can distinguish complex forms and take account of order and distance.

4. Stage I. Recognition of familiar objects, then of topological but not of euclidean shapes.

Although the child can recognize familiar things, he cannot abstract shape for want of sufficient exploration. When he can abstract shape he begins with topological relations only. Why are the exploratory movements absent, when nothing would appear easier than to follow the contour with one's finger? What is the abstraction of shape, and why are topological relations simpler?

Based upon a chance indication, he likens the shape touched to a visual shape possessing the same characteristic, not bothering about the rest of the object or attempting to put together the total structure. E.g., the triangle is identified with the square because of one of its angles, the ellipse with the notched semicircle on account of its curved outline, the triangle with the notched semicircle because of one of its points, etc. These errors are due to inadequate exploration. Why? The distinction between perception and perceptual activity becomes important. Perceptual activity begins with "decentration." Touching any one part involves centration and an initial tactile perception. Touching another portion involves another. Each leads to the over-emphasis of the part contacted. To pass from one centration to another, which coordinates and corrects the perception, implies an activity to some extent motor in character. The lack of exploration may be explained as the result of a general deficiency in perceptual activity itself. The child's perceptions are still passive or static instead of being integrated in a system of sensorimotor coordinations. Exploration by eye is probably simpler and takes in more things at once than exploration by touch, and this is perhaps why visual shapes are more easily reconstructed than tactile ones. We may now understand more about "abstraction." Shape is not just inherent in the object, since it also results from the subject's movements in handling the object. Only because these movements are coordinated from their inception is the shape grasped as a single whole, and not as a series of discrete elements put together as an afterthought. Now we have seen that only topological relations are noticed. They follow from the natural procedures of tactile exploration, following step by step -- proximity, separation and surrounding are detected. These relations provide the distinction between open and closed figures.

Stage II. Beginning of Euclidean shapes. Now the child notices the difference between curved and straight lines, but he does not distinguish sizes of the shapes. (Examples, pp. 28-29). Exploration is more active, but it is haphazard -- the child happens upon some cues and uses these to name the figure, disregarding additional features. E.g., any figure with an acute angle, such as a notched semicircle, might be called a triangle if he happened to feel a point first.

Now he begins to discriminate rectilinear and curved shapes on the basis of angles. Piaget gives an example of this in which a child draws all the rectilinear shapes as a single closed shape with one right angle.

It is analysis of the angle which marks the transition from topological to Euclidean space. It is not the straight line itself, but the angle. Visual recognition of an angle may be a multisensory process (e.g., he exclaims, "It pricks!"). To reconstruct an angle as a pair of lines, the child must coordinate actions of eye and hand. This is the only way to get beyond simple "It pricks!". We shall find both here and in drawing, that he represents not so much the model directly, as the perceptual activity itself.

The tactile rhombus (as well as the visual rhombus) -- Piaget means a rhombus oriented as a diamond -- is a difficult figure to represent. Gestalt theory would consider its symmetry a good form. But here it makes difficulty. The rhombus is drawn as a square or rectangle, adorned with a point or triangle or "wire" ("it's a trolleybus") or ears for obtuse angles, square with diagonal, etc. One finds the same results when the child is asked to draw a visually observed rhombus.

Only at about seven years do reversible operations enable correct representation. Prior to this there is exploration, but it is lacking in operational guidance -- he keeps moving ahead all the time. Example of rhombus, p.35 -- the child says of each side in turn, "It's leaning, it's leaning, it's leaning, and this one is leaning too," but cannot coordinate this information properly. Because he is moving ahead and not returning to coordinate details, he may represent a swastika as a staircase or a branched structure as in the accompanying figure.

Stage III. Beginning of reversible operations. Now he begins to group explorations in a common plan, with a fixed reference point. Examples here and in Werner, H. Comparative Psychology of Mental Development of the coordination and articulation of elements of a drawing.

Conclusions

Piaget tries to distinguish between "pure perception" involved in recognition and "perceptual activity" involved in reconstruction. Recognition is instantaneous (at least after a few months). Pure perception remains constant, while perceptual activity develops with age.

The image is the transition from perception to mental representation, and involves motor activity, a product of imitation.

CHAP. II: TREATMENT OF ELEMENTARY SPATIAL RELATIONSHIPS IN DRAWING. "PICTORIAL SPACE."

Does the reconstruction of conceptual space pass through the same phases as does construction of perceptual space? The child has at his disposal the achievements of perceptual activity and sensorimotor intelligence. In terms of perception he knows distances, straight lines, squares, and perspective. Nevertheless, he is unable as yet to translate these entities into thought or imagination the moment his efforts are no longer supplemented by direct perception. Despite possible drawbacks and problems of interpretation, drawings, when checked by other methods, provide valuable material for analysis and separation of the perceptual and representational components.

Section 1: Space in spontaneous drawings.

The three principal stages in children's drawings once the level of mere scribbling is left behind are well known and have been termed by Luquet (*Le Dessin Enfantin*, p. 154, 1927): (1) synthetic incapacity, (2) intellectual realism, and (3) visual realism.

1. Stage I. Synthetic incapacity.

A 3 1/2 year old might draw a man in the shape of a large head to which are appended four strokes for arms and legs, plus a small trunk separate from the limbs. The head contains two eyes, a nose, and a mouth, but the latter is placed above the former. How do we interpret this? A drawing is a representation, which means that it implies the construction of an image, which is something altogether different from perception itself, and there is no evidence that spatial relationships in composing this image are the same as those in the corresponding perception. A child may see the nose above the mouth, but when he tries to conjure up these elements and is no longer really perceiving them, he is liable to reverse their order, not simply from want of skill in drawing or a lack of attention, but from the inadequacy of his instruments of spatial representation which are required to reconstruct the order along the vertical axis.

The stage of synthetic incapacity is a representation of space which neglects euclidean relationships like proportions and distances, and perspective relationships, and which has barely begun to represent the simplest topological relationships. Let us consider the latter:

1. Proximity. The most elementary relation plays a part in every drawing that goes beyond mere scribbling. For example, in the construction of a face the various parts are drawn near to each other and not dispersed to the four corners of the sheet of paper. But in a complex figure, such as the representation of a man, while proximity is taken into account in a general way, it is not adhered to in detail. (E.g., attaching the arms and legs to the head, while drawing the trunk separately; fingers attached to arms; dog's tail attached to his head.)

2. Separation. In Sec. III on drawing simple geometrical shapes we shall see the difficulty in separating elements represented in global fashion.

3. Order. Since, as we shall see, this relation constitutes the synthesis of proximity and separation, it is natural that it has barely begun to be established at this stage. At the most, pairs are correct. We shall find in Chap. III that before the age of four a series of multi-colored beads is reproduced not in its original order, but simply as isolated elements.

4. Surrounding, or enclosure. These relationships are the most clearly indicated in the case of simple shapes (we shall see this in the example of small circles put inside, outside, or on the contour or closed curved figures), but errors frequently arise when complex shapes are involved. Thus eyes are put outside the face, buttons to one side of the body, the roof projecting into a house. Further difficulties will be seen in the study of knots (Ch. IV).

5. Continuity and discontinuity. While visible in broad outline, they are still primitive in more complex shapes. One of the principal features of synthetic incapacity is that parts of figures are simply juxtaposed instead of being continuously linked together. Thus a rider remains suspended above his horse, a hat above a head, etc. (See H. Werner, Comparative Psychology of Mental Development for numberous examples and drawings of such lack of articulation.)

Thus primitive topological relations appear, but fail to be generalized to complex structures. Without altogether excluding the part played by technical lack of proficiency, we shall see that the predominance of topological relations really represents laws of pictorial space.

2. Stage II. Intellectual realism.

The child draws not what he actually sees of the object, but "everything that is there." Consequently there is no question of want of technical skill or lack of attention. Euclidean relations are barely beginning to emerge, while the elementary topological relations are universally applied to all situations. (1) Proximities are correct, or at least aimed at; arms and legs are attached to the trunk, the eyes placed in the head, and the two eyes are always placed side by side, even in profiles, where proximity runs counter to perspective. (2) Separations are made more clearly, and there is some progress in analyzing the separate elements. (3) An order of succession is found in complex drawings (landscapes, houses, etc.) which may not be in accordance with each dimension of a system of coordinates, but which does follow a direction corresponding to a practically possible order. (4) Surrounding assumes very great importance, since in many drawings the interior of things is represented by means of transparency, (food in the stomach, a duck in its egg, potatoes in the ground). (5) Continuity is well defined, as contrasted to the purely external justapositions of Stage I.

However, there are neither coordinated points of view nor any general coordinates. While intellectual realism derives its elements from concepts which are just becoming projective and euclidean, nevertheless, the relations are mainly topological, of the above five types. At this level we find the tentative beginnings of the accurate copying of euclidean shapes, and a start made in the construction of projective relationships (Chap. VI on the straight line) but as yet no coordination of perspective in the drawing as a whole (Chap. VIII), no proportions, no coordinate systems (Chap. XIII) applicable to a complex layout (Chap. XIV).

3. Stage III. Visual realism.

About 8-9 years of age there appears a type of drawing which tries to take perspective, proportions, and distance into account all at once. The lateness indicates that projective and euclidean notions are slow to appear in the realm of representation, in contrast to their development in perception. Projective and euclidean relationships will be found to develop concurrently and interdependently. These relations, in contrast to the topological relations, deal with relations between figures, and lead to the development of comprehensive systems.

Section 2: The drawing of geometrical figures.

Technique and general results.

After the child has drawn a man from memory, he is asked to copy the geometrical figures shown in the page of illustrations. These include an irregular shape with small circle outside, inside, or on the boundary, circle, intersecting circles, triangle, square, triangles in circle, ellipse, rhombus (diamond), two circles, vertical or slanting cross. The children were also asked to reconstruct the straight-sided models using matchsticks. Children between 2 and 7 years of age, although familiar with euclidean figures such as circles, squares, etc., did not primarily express in their drawings the perceptual features of "good Gestalt" which such figures present, but rather the topological characteristics of proximity, closure, surrounding, etc., as already seen in the case of haptic perception.

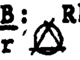
Stage 0: no purpose or aim can be discerned in the drawings. They are simply simply scribbles (see figure) which show no variation whatever the model (up to the age of 2/6-2/11).

Stage I, substage A: (to 3/6-3/10) Scribbles appear to vary according to the model being copied, open shapes being distinguished from closed ones (figure).

Substage IB: (average 3/6-4 years) One can begin to speak of real drawings, though it is only topological relationships which are indicated with any degree of accuracy, euclidean relationships being completely ignored. Thus the circle is drawn as an irregular closed curve, and so are squares and triangles. They are all drawn as closed curves with perhaps an occasional symbolic suggestion, such as wavy lines jutting out to indicate angles. Only the open figures are distinguished from these, like the cross (as two or more intersecting lines, not necessarily straight). No distinction between straight and curved figures, but correct representation of topological properties of the three figures consisting of the irregular closed curve with a small circle in, out, or on.

Stage II: (starts at about 4) Progressive differentiation of euclidean shapes. Between IB and IIA curved shapes begin to be distinguished from straight-sided ones, though the latter remain undifferentiated from each other (notably square & triangle), so that these figures are given straight sides with no regard to how many. It is most frequently that the rectangle is reproduced correctly.

Substage IIA: Shapes are gradually distinguished according to their angles, and even their dimensions. Distinctions made between square-triangle, circle-ellipse. Squares and rhombuses with diagonals are ok (⧄ , ◈) though not the ordinary rhombus (◇). The vertical and slanting crosses are distinguished. The circumscribed figures have the proper shapes, but their points of contact are incorrect, while the contiguous circles are ok.

Substage IIB: Rhombus correct, and circumscribed figures are gradually mastered, except for  , in which circle and triangle cross three times.

Stage III: (6/6-7 years). All the problems are overcome.

Discussion of the stages. (Reading the protocols in the book is essential.)

Stage 0, simple rhythmic movements. Stage I, beginnings of discrimination (IA), then appearance of closed curved shapes (IB).

Although the child cannot copy any figure at this stage (O), his scribbles, whether spontaneous or resulting from a fruitless attempt to copy a shape, are important for the psychology of spatial representation. This is especially true if shape is abstracted from the subject's own movements rather than from the object which occasions them. The primary feature of the children's drawings of scribbles is their simple rhythm, the continual movement of the hand about the paper, and it is from such a rhythmic pattern that the first shapes come to be distinguished at Stage I. Every mental mechanism passes from rhythm to systems like groups by means of regulatory processes which begin by coordinating the component parts of the initial rhythms into systems of reversible operations. The rhythmic movements already contain in an undifferentiated state all those elements which will later go to make up the drawing of straight lines, curves, and angles, even though the child cannot yet extract or "abstract" these from the rhythmic complex. One could extract pieces of lines, angles, ellipses, and crude circles. With a little guidance and practice the child can reproduce at least some aspect of the model.

One must try to draw a distinction between the way in which the child appears to divide the perceived figures into open or closed shapes, and the manner in which he tries to interrupt the rhythm of his own continuous movements in attempting to express the different features of each model. These do not necessarily coincide because in drawing a single closed shape he must interrupt his motor rhythm and is thereby subject to an involuntary discontinuity. The problem amounts to producing a given shape through rhythmic movements which tend to oscillate between vague zig-zags and curved paths. He must therefore break this continuous rhythm to draw a circle, while at the same time taking advantage of its bends and natural closures. Thus in some of the examples the circle is formed from a number of intertwined spirals which do not meet properly to close the circle. But the failure of closure is one of skill, since there is a huge difference between these and figures in which closure is not aimed at. Slight differences appear in the movements used to compose squares as opposed to circles. Thus there is an effort to reproduce the figures by extracting parts of his rhythmic movements. The extracted features are primarily topological: open or closed, whether there is anything inside or outside the closed contour. This conclusion probably holds for spontaneous drawing as well.

Piaget's protocols show the process of arresting or interrupting the rhythms of scribbling: breaking them down into discrete elements, arranging these elements in relation to one another, and then reassembling them with a series of perceptual-motor regulations. Two curved lines or part of a spiral may be used to form a square or circle. In each instance the interrupted rhythm is superseded by a series of individual movements, adjusted somewhat to fit each other. Since these children are as yet incapable of performing reversible operations of thought, the present precursors of these mental operations must be related directly to the perceptual-motor images. And the patterns most easily represented by perceptual-motor activity are the topological ones. Is the failure to distinguish rectangles and triangles from circles and ellipses one of simple motor difficulty? For it might be that the circle corresponds to a single natural movement, while squares require the deliberate adjustment of lines in various directions and locations. But the protocols show circles made by the adjustment of two separate curves, and spontaneous scribbles show as many nearly straight lines as curved lines. One child can draw a pine tree with a trunk and branches at right angles, and thus disposes of all he needs to construct a square, which he is nevertheless unable to copy. Equally complicated problems of deliberate combination can be mastered when they involve only topological relations (putting small circle astride boundary of closed shape). Thus the problem is not one which depends upon mere motor ability, but rather on the method of composition itself -- on the type of regulatory mechanism which will result in the construction of a shape on the basis of elements isolated from the original pattern.

Thus it comes about that as soon as the rhythmic movement has been broken down into discrete elements, the very fact of connecting or not connecting them together results in relationships of proximity and separation, enclosure and openness, ordered succession and continuity. In short, topological relations, (as opposed to relations involving directions, angles, straight vs. curved lines) are first in order of appearance because they are inherent in the simplest possible ordering or organization of the actions from which shape is abstracted.

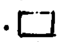
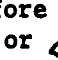
The upright cross is drawn successfully, while circles and squares are not distinguished. This is because the cross is just an open shape consisting of two lines. All the closed shapes are lumped together. It is thus wrong to say the child represents the circle better than the square, or that the square is likened to a circle. Rather, children at this stage are not influenced by the metrical and projective properties. The triangle is likewise merely a closed shape, although lines and points may be added to mark the angles. One child starts the cross in the same way-- an ellipse with long wavy lines -- before drawing it as an open figure with crossing lines. The inscribed figures are also drawn on topological principles. In the case of the two circles, contiguity is frequently indicated by one or two strokes connecting separate closed shapes. (See H. Werner, Comparative Psychology of Mental Development.) But the two large circles in this example may be more difficult than the large shape with small shape in, out, or on because the two large circles may be perceived as two separate wholes rather than as a single whole.

In Substage IB pictorial representation still does not correspond to the perceptual data at this level, for the latter have long since acquired a projective and euclidean character. Far from being something determined by perceptual "good Gestalt," pictorial representation expresses the basic requirements for the composition of figures-- the active rather than the perceptual aspect of their construction. The "abstraction of shapes" is not carried out solely on the basis of objects perceived as such, but is based to a far greater extent on the actions which enable objects to be built up in terms of their spatial structure. This is why the first shapes to be abstracted are topological, for these relations express the simplest relations in the coordination of dissociated motor rhythms.

Stage II. Differentiation of euclidean shapes.

Criterion for appearance of Stage II is the successful reproduction of the square or rectangle (the latter usually comes slightly earlier). Binet and Terman showed that copying a square was a test for a mental age of 4, and this is quickly followed by the triangle (or the triangle may come first). They found that the rhombus was not mastered until 6 or 7 years.

As did Stage I, Stage II likewise parallels haptic perception. The transition from IB to IIA is marked by the simple discrimination between curved and straight lines within more or less curved structures, while there is no clear discrimination between squares and triangles. (See the protocols.) How are angles and rectilinear shapes derived from closed curved shapes, and why is there a relative lack of differentiation between the different rectilinear shapes?

In relation to angles, the results of drawing confirm the results of haptic perception: it is the analysis of angles which leads to the discovery of straight lines rather than the other way about. The earliest squares or triangles are simply circles distinguished by the addition of one or two angles. Now before being obtained by adjusting discontinuous lines (e.g.  represented as  or ), the angle is formed either by means of transforming an arc into a kind of blunted point, or by a rapid out and return movement with a space between the two paths, resulting in a sort of beak found in spontaneous scribbling. The angle is thus extracted from the incipient rhythmical movements, as are the circular shapes themselves, and it is only through this process of extraction that the straight line is differentiated as such. However, at a later stage the angle can be reconstructed in turn by the intersection of two straight lines. It is just because this adjustment of straight lines drawn alone is so much more difficult than the abstraction of a complete angle, that the first rectilinear shapes are so little differentiated from each other. In order to connect two given straight lines one must take into account inclinations, number of lines, points of conjunction, and distances. That is why the rectangle may be formed by two lines which each contain a right angle. Many of these curved angles will not fit together with the other lines in the drawing. The coordinations necessary to this begin to take place during IIA as a result of various detail adjustments concerned mainly with sizes and inclinations of lines, giving rise to differentiation of the square, triangle, and rectangle from each other, and the circle from the ellipse. (See protocols.)

Stage I consisted mainly in the process of abstraction of elements of the rhythmic scribbling. Stage II, on the other hand, begins to be concerned with methods of composition starting with the abstracted elements. But these methods are not performed by means of reversible operations, as they will be from Stage II on, but only by a series of tentative adjustments governed by perceptual-motor regulatory mechanisms, of which three may be mentioned. (a) Length and distance. These are long since known in perception, but only occur in drawing after a distinction has been drawn between curved and rectilinear figures. This is not merely due to difficulty in drawing, for the same result was found in IIA of haptic perception. (b) Contiguity and separation. So long as only indefinite closed shapes were involved we saw, in relation to boundaries and enclosures, how precocious were these topological relations. But no sooner is an attempt made to relate them to definite metrical shapes than they have to be reconstructed on the euclidean level. We have already seen this in the example of the two large circles, and in the various combinations of inscribed figures. (c) Inclinations. These were previously unknown, but are essential for the discrimination of triangle-square, upright-slanting crosses, and construction of diagonals and the rhombus. Up to IIA these are impossible--he cannot distinguish triangle and square. Inclinations gradually begin to be mastered at about 4/6-5/6. However, when it is a question of two completely separate figures, ability to judge their inclination (e.g., to estimate horizontals by finding some point of reference outside the figures themselves) is never acquired prior to the appearance of reversible operations, and such problems are only mastered after the age of seven. The problems of the present stage are purely internal to a single figure, for example, the inclination of one side of a triangle to the other two. (The two crosses are separate but closely related figures.) The fact that 2 or 3 years work is required to pass from copying the square to copying the rhombus shows that to construct an euclidean shape requires more than a correct visual impression. The same difficulties occur in matchstick construction of the rhombus. See the illustrations of the stages in the construction of the rhombus. IA: wandering open line. IB: closed curve, occasionally with a thread-like appendage denoting an acute angle. IB-IIA: angles and straight sides, though the slope cannot be controlled. Thus it is confused with square or triangle. Acute angle suggested by line at the corner, a triangle (hats, beaks, etc.) placed over the square. Obtuse angles may be indicated by little triangles or even curves (ears, etc.). (See illustration.) IIA: begins to represent the inclination itself, obtaining various trapezoids, pentagons, etc. Figures may remain open for lack of being able to close them in a way consistent with the desired inclination. The missing factor is symmetry, which necessitates the inversion of order. "It must be like the top, but I can't manage it!" IIB: rhombus is at last conquered, and problems of contact and separation, external and internal in circumscribed figures are also resolved, though still by adjustments rather than by instantaneous organization. In the successive approximations, the analogy with exploration in haptic perception is evident.

Stage III and Conclusions

Stage III of haptic perception started at the point where the movements through which the shape is abstracted could be defined as operational, i.e., flexible and reversible enough to return constantly to the point of reference on which the subsequent construction is based. While this distinction does not show itself very clearly in drawing, except in the case of figures like the swastika, it is noticeable that from the age of 7, many children can draw all the models correctly right away, their construction presumably being anticipated by a mental image drawn up in advance (mainly in terms of potential measurements, coordinations, etc., as may be seen in Chap XIV on diagrammatic layouts, such as the plan of a village). At the present stage the analysis of geometrical figures is more developed than that of freely drawn figures like that of the mannikin.

In summary, geometrical space is not simply a "tracing" made over a simultaneously developed physical space, temporarily corresponding with in point for point. The abstraction of shape actually involves a complete reconstruction of physical space, made on the basis of the subject's own actions, and to that extent, based originally upon a sensori-motor, and ultimately on a mental, representational space determined by the coordination of these actions. This is the origin of all the geometrical relations in drawing. The matchstick experiments provide confirmation. Matches emphasize constructional and adjustment aspects. Adjusting 3 matches to form a triangle is just as hard as drawing it, and the two problems are solved at the same time. The general conclusion is that reconstruction of shape is not just isolation of shapes and perceptual qualities, but an active process of putting in relation, and it is based on the child's own actions.

CHAPTER III: LINEAR AND CIRCULAR ORDER (selected topics)

Piaget studies five main tasks of representation:

1. Reproduction of simple linear order--colored beads on rod, dresses on line.
2. Transposition of circular order into simple linear order.
3. Establishment of reverse order.
4. Direct or reverse order of stacking (take from line and stack in basket--could the red one go on top?).
5. Transposition of beads in figure "8" order into circular or linear order. (Similarity to problem of recognizing simple overhead knot in various degrees of tightness.)

Stage 0: No correspondence; any objects placed in any order (2-3 years).

Stage I: Simple correspondence of types of objects (i.e., uses the right objects, but no regard paid to order. (Stage IA, 3-4 years.) Toward the end of this period (Stage IB), the child can order pairs of elements by proximity, but cannot coordinate such pairs with each other to achieve correct order in the large.

Stage II: (4-6 years): Child is capable of making ordered correspondences.

IIA: Confined to objects which are face to face; and various perceptual features acting as controls are noticeable, bringing out the reason for this limitation. That is, the immediate perceptual configuration must be such as to make the proper correspondences stand out in one glance. There is no comprehension of "between."

IIB: No longer requires copy and model to have identical perceptual configurations, and as a result can transpose circular into linear order, but cannot establish reverse order. Between IIB and III can establish reverse order, but only through trial and error. Transposing figure "8" to linear order is still too hard.

Stage III: Beginning of reversible operations. Reverse order still not made immediately. Figure "8"--linear still slightly difficult, but overcome.

Piaget's examples of reactions to the problem give an idea of the psychological conditions necessary for the construction of a sequence. If it is desired to reproduce exactly a series of adjoining elements:

(a) Proximities of the elements must be preserved.
(b) Proximities must be regarded as independent of size of intervals, and especially as independent of perceptual proximity.
(c) The operation of separation must be reversibly performable (see below).
(d) It must be possible to maintain a constant direction of travel.

Presupposed by the above, but absent in Stage 0, is the ability to find identical elements on the basis of properties.

Stage I cannot get all proximities right--he tries to correct one and spoils others. Similarly, in drawing, he preserves proximity for simple shapes, but when he tries to draw complex things, he attaches arms to head, etc. In both drawing and making ordered series, proximity is the most primitive of all relations. Nevertheless, separating nearby elements in the course of reproducing them in a drawing or copying them in a series is enough to alter the proximities themselves. This means that the original relations must be recreated after having been broken down. But this resynthesis presupposes a fixed order of succession for the original analysis. But this fixed order can only be maintained by coordinating the proximities between the separated elements. Thus there is a vicious circle.

He has no other criterion of order and fixed direction except proximities. Given a pair AB, he is just as likely to put it BA because he cannot judge their orientation. He must achieve motor coordination sufficient to enable him to keep to one direction. To arrive at this, he must proceed step by step, and thus rely upon coordination of proximities. Proximity, separation, and sense of direction become linked up in the next stage.

In Stage II the child achieves the intuitive representation of order when the correspondence is visible. The sense of direction is guaranteed by organizing movements according to a simple perceptual pattern. He cannot do it if the pattern is destroyed -- e.g., when he must change circular to linear order. Moreover, a circular clothes line has no points of refernce. (One child can do the task with reference to fixed points regarded as "laundry poles.") The less advanced children cannot follow the correspondence between beads on necklace and rod even with their fingers, nor can they establish correspondence between two necklaces by eye.

In Stage IIB the child can transpose circular to linear order, but cannot place beads in reverse order. Thus there is still no complete mobility of mental operations. He cannot solve the stacking problem because it involves a 90° rotation of the ordered line. Relations cannot be maintained invariant. Given several necklaces, identical except that one is in reverse order, the child may say that all are identical. He says they are the same because red is next to green in both, etc. But he regards as different two linear arrangements in reverse order, because the two orders lead to different reference points. We shall see in the course of these studies that it is inadequacy of motor coordination which tends to confuse the two directions. We see the same thing in the problem of coordinating perspective viewpoints and in other problems considered by Piaget and Inhelder. Motor skills are vital to the development of the mental representation of space.

Between IIB and III the child achieves some of the coordinations by trial and error. In Stage III he begins to achieve correspondence of orders based upon reversible operations. The book goes on to discuss this and later stages, and to present conclusions.

A principal conclusion is that there is a difference between perception and representation of order. Representation means reproduction of order, either as a mental image or as a material copy. Perception of order means no more than perception of proximities, separation of neighboring elements, and constant direction of travel in successive centrations.

In reassembly after breaking up the perceptual configuration into its elements, the direction of travel becomes important. It is maintained at the end of Stage I and during II and III by movement. In IIA it is the act of running the length of the model by eye or with fingers, and mentally transferring the separated elements. In III mobile mental operations take over.

Thus order is no more the product of abstraction direct from the object than are the elementary topological relations. Order is abstracted through coordination of actions such as transferring (transporting objects mentally) and replacing piece by piece. It is the result of reconstructing objects through ordered actions, not a directly abstracted quality. The physical order found in the object is reproduced through the adaptation of these actions.

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